## Reduced model analysis of supercritical and subcritical chirping Alfvén eigenmodes

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We present a new method for analyzing fundamental kinetic plasma parameters such as the linear drive and the external damping rate based on experimental observations of chirping Alfvén Eigenmodes (AEs). The method consists of fitting procedures of nonlinear chirping characteristics between the so-called Berk-Breizman (BB) model [1] and the experiment. The method is validated through comparisons of kinetic plasma parameters against former analyses on the Toroidicity induced Alfvén Eigenmode (TAE) on MAST [2]. Major advantages for this technique are 1. fitting conditions estimated only from the spectrogram of the magnetic fluctuations, and 2. unified treatment of supercritical and subcritical AEs. The apparent contradiction between the linear theory and the behavior of subcritical solutions observed in simulations is clarified.



Figure 1. Spectrogram of magnetic perturbations featuring a series of chirping TAEs (a), and simulation of the BB model (b).

The BB problem is a generalization of the bump-on-tail problem, where we take into account a collision term that particle annihilation represents and injection processes at a rate  $v_a$ , and an external wave damping accounting for background dissipative mechanisms at a rate  $\gamma_d$ . Chirping is a special kind of chaotic behavior where the mode frequency sweeps for a life time  $\approx v_a^{-1}$ . In this study, we fit the spectrogram of chirping simulations of the BB model to the spectrogram of magnetic perturbations measured in MAST shot #5658, in terms of the chirping velocity, chirping amplitude, and chirping period, where we normalize time with the mode frequency. The comparison between the experiment and a reduced simulation, in Figure 1, shows a qualitative agreement for these chirping characteristics. However, the experiment presents phases between major chirping events that are much more quiescent than in our simulations. This observation raises the question of the applicability of the BB model to the experiment. In particular, it is still an open question if a bump-on-tail is an appropriate model for the actual energetic particle distribution in the experiment.

In this example, the parameters are found in the supercritical regime, but the same analysis can be applied to possible subcritical TAEs. When the initial perturbation amplitude is large enough, chirping solutions are observed in a regime where the linear theory predicts a negative growth rate [3],  $\gamma < 0$ , where  $\gamma \sim \gamma_L - \gamma_d$  is the linear growth rate including the contributions of external damping and collisions, and  $\gamma_L$  is the linear growth rate in the collisionless case without external damping. The discrepancy between the linear prediction and the evolution of the wave can be understood by examining the electric field evolution equations:

$$\frac{d\omega_B^2}{dt} = (\gamma + \varepsilon)\omega_B^2 \equiv \gamma_{NL}\omega_B^2$$
(1)

$$\varepsilon = \frac{4\pi e^2 \omega}{2mk} \Re \int_0^t dt' \omega_B^2(t') \int_{-\infty}^{+\infty} e^{iv(t-t')} \frac{\partial(\bar{f} - f_0)}{\partial v}(v,t') dv$$
(2)

where  $\omega_B$  is the bounce frequency of deeply trapped particles, *e* and *m* are the electronic charge and mass,  $\bar{f}$  is the spatial average of the distribution function and  $f_0$  its initial value. When



Figure 2. Simulation of the BB model in the subcritical regime (blue), and nonlinear growth rate (green). The linear theory (red) is valid until around  $\gamma_L t = 10$ . After around  $\gamma_L t = 50$  the spreading of phase space structures prohibits the estimation of the nonlinear growth rate with the same method.

the correction  $\varepsilon$ , induced by small perturbations of the velocity distribution, exceeds  $|\gamma|$ , the amplitude of the wave grows. In Figure 2, the nonlinear growth rate  $\gamma_{NL}$ , estimated by averaging the slope of the velocity distribution in the neighborhood of the resonant velocity, is observed to become positive.

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