Dynamics of Structures in Configuration Space and Phase Space: An Introductory Tutorial

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Abstract

Some basic ideas relevant to the dynamics of phase space and real space structures are presented in a pedagogical fashion. We focus on three paradigmatic examples, namely; G.I. Taylor's structure based re-formulation of Rayleigh's stability criterion and its implications for zonal flow momentum balance relations; Dupree's mechanism for nonlinear current driven ion acoustic instability and its implication for anomalous resistivity; and the dynamics of structures in drift and gyrokinetic turbulence and their relation to zonal flow physics. We briefly survey the extension of mean field theory to calculate evolution in the presence of localized structures for regimes where Kubo number $K \simeq 1$ rather than $K \ll 1$, as is usual for quasilinear theory.

"We all know that the real reason universities have students is in order to educate the professors."

John Archibald Wheeler

"Reward the sneezers who stand up and spread the ideas."

Seth Godin, on viral marketing

I. INTRODUCTION AND BASIC CONSIDERATIONS

The aim of this tutorial is to introduce the ways we think about and describe relaxation, transport and instability in a system consisting of an ensemble of localized structures. Indeed, in simple terms, turbulence is usually better thought of as a 'soup' or 'stew' of 'eddys,' 'vorticies,' 'blobs,' 'clumps,' 'holes,' etc. than as an ensemble of (very) weakly nonlinear waves. However, in MFE, we almost always *calculate* using quasilinear theory, which is based on the idea of turbulence as a random ensemble of waves. We follow this familiar recipe in spite of the fact that at the same time, we often invoke or tacitly assume that the strength of saturated turbulence given by mixing length guesstimates[1], which posit that $\tilde{v} \sim \Delta_c/\tau_c$. Here $\tilde{v}$ is a typical fluctuating velocity, Δ_c is the correlation scale and τ_c is the correlation time. Of course, such a mixing length criteria is simply another way to write Kubo number $K \simeq 1$, where $K = \tilde{v}\tau_c/\Delta_c$. $K \simeq 1$ implies that the particle or fluid element rotation time in a vortex structure is comparable to the lifetime of that structure, while quasilinear theory assumes the lifetime is short compared to the rotation time (i.e. $\tau_{ac} < \tau_b$). Thus, it is difficult to see how $K \simeq 1$ and quasilinear theory can be mutually compatible, despite perpetual claims to the contrary.

More generally, a structure such as an 'eddy' or 'blob' can be distinguished from a wave in that an eddy does *not* correspond to the zero of a collective response function (dielectric), i.e. $\epsilon(k, \omega) = 0$, while a wave does. MFE theory is obsessed with the zoology of linear waves and instabilities in spite of the fact that, as noted above, concepts derived from localized structures are frequently more germane to the true, physical dynamics of the real turbulent state. Structures are usually thought to form in the process of nonlinear saturation of

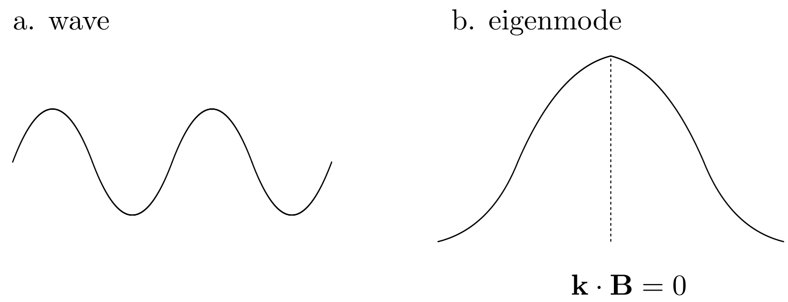

FIG. 1. wave and eigenmode

instability. However, as we will discuss, localized structures are sometimes more efficient in *tapping* free energy than linear waves are, and so structures can also be a mechanism for *triggering* instability and relaxation. We will see that this property follows from the fact that structures scatter energy and momentum differently than waves do, on account of the fact the structure is *localized* in phase space. In the particular case of drift wave turbulence, consideration of potential vorticity structures provides an illuminating alternative route to understanding zonal flow generation via the conservation of momentum in drift wave-zonal flow interaction.

Given the aphorism that "a picture is worth a thousand words" - and at least as many equations - we deem it useful present a few relevant cartoons here, so as to put flesh on the rather abstract concepts we have been discussing. Figure (1a) shows a wave and (1b) shows an eigenmode localized at a $\mathbf{k} \cdot \mathbf{B} = 0$ surface. These should be compared to cascading eddys, shown in Figure (2a) or an isolated coherent vortex, shown in Figure (2b). Also localized structures can interact with mean profiles. One example is shown in Figure (3), where a localized gradient relaxation event (i.e. gradient flattening) generates a blob (i.e. local quantity excess) propagating down the gradient and a void (i.e. local quantity deficit) propagating up the gradient. Structures can form in phase space, as well. Figure (4) shows a phase space density hole, in velocity space. Since total phase space density must be conserved, the hole perturbation can grow if the centroid of the hole moves up the gradient. We will refer back to these cartoons from time to time in this paper.

The reminder of this tutorial is organized as follows. In section II, we discuss the Rayleigh

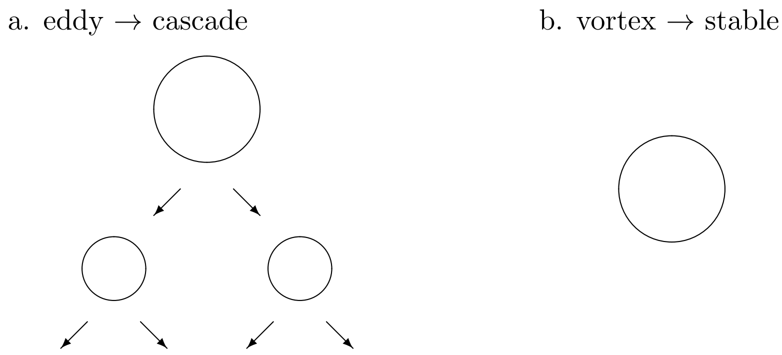

FIG. 2. eddy and vortex

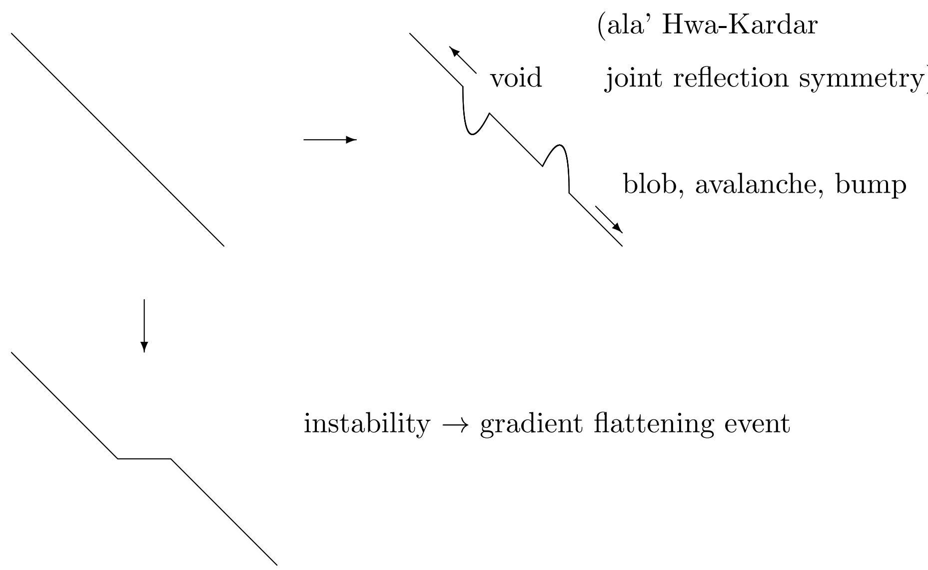

FIG. 3. void and bump

inflection point theorem. After reviewing the classic modal approach we present G.I. Taylor's far more physical and intuitive derivation based on considering the displacement of a localized PV blob. As a bonus, from there we immediately derive basic insights into zonal flow generation and the content of the Charney-Drazin non-acceleration theorem for wave-mean flow interaction. In section III, we re-visit the current driven ion acoustic problem, and discuss a nonlinear, structure-based instability mechanism which is complementary to the text book example based on linear theory. We cast this story in terms of growth of a granulation in phase space. The instability mechanism is nonlinear. We also consider the growth of localized structures in the context of the new classic paradigm of the Berk-

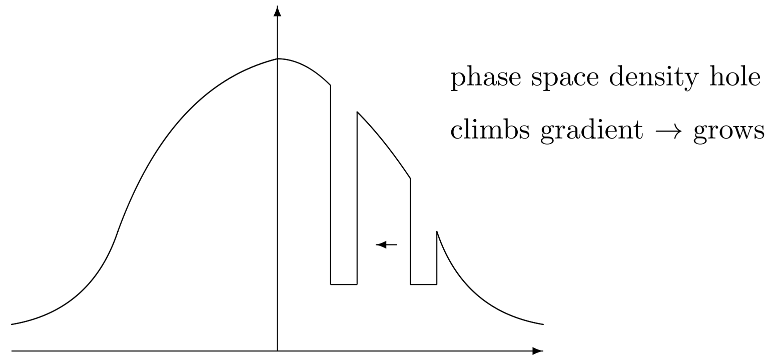

FIG. 4. hole in phase space

Breizman model. In particular, we suggest the possibility of a new nonlinear instability for that model. In section IV, we discuss phase space density structures in the context of drift and gyrokinetic turbulence. That section to some degree unifies the discussions in Section II and Section III. Special attention is given to the relation of structure dynamics in kinetic drift-zonal flow systems to the Charney-Drazin non-acceleration theorem discussed in section II. In Section V, we briefly outline the Dupree-Lenard-Balescu theory for calculating transport in the presence of phase space density granulations, where Kubo number $K \sim 1$. That section is short, as the subject is discussed at length in other recent tutorials. Section VI is a brief concluding comment.

II. RAYLEIGH CRITERION, POTENTIAL VORTICITY, AND ZONAL FLOW MOMENTUM

In this section, we first review the traditional 'modal' approach to the Rayleigh criterion, then present a far more physical, 'structure based' approach first given by G.I. Taylor. We then extend this line of thought to extract the essence of several key relations between wave and zonal flow momentum.

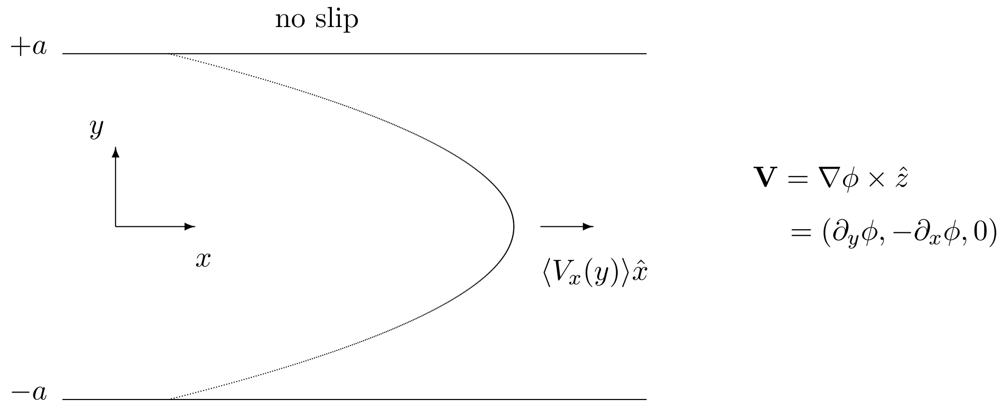

FIG. 5. Flow configuration

A. Something Old: Modal Derivation of Rayleigh Criterion

The time honored Rayleigh criterion[2–4], a necessary condition for inviscid shear flow stability, is relevant to the flow configuration shown in Fig. 5.

Here $q = \nabla\phi^2$ is the potential vorticity, so PV conservation is just $d\nabla^2\phi/dt = 0$. Straight forward linearization then gives the Rayleigh equation for eigenmodes of the shear flow:

$$\left(\partial_y - k_x^2\right)\tilde{\phi}_k + \frac{k_x\partial_y\langle q\rangle}{\omega - k_x V_x(y)}\tilde{\phi}_k = 0 \tag{1}$$

Multiplying by $\tilde{\phi}_k^*$, integrating from $-a$ to a, noting the no-slip boundary condition, and writing $\omega = \omega_r + i\gamma_k$ explicitly, then gives

$$-\int_{-a}^{a} dy \left(|\partial_y\tilde{\phi}_k|^2 + k_x^2|\tilde{\phi}_k|\right) + \int_{-a}^{a} dy \frac{k_x\partial_y\langle q\rangle\{(\omega_r - k_x V_x(y)) - i\gamma_k\}}{(\omega_r - k_x V_x(y))^2 + \gamma_k^2}|\tilde{\phi}_k|^2 = 0 \tag{2}$$

Eq. (2) is a complex equation, so both the real and imaginary parts of the lefthand side must vanish. For the imaginary part, we have

$$\gamma_k \int_{-a}^{a} dy \frac{k_x\partial_y\langle q\rangle}{(\omega_r - k_x V_x(y))^2 + \gamma_k^2}|\tilde{\phi}_k|^2 = 0 \tag{3}$$

Thus, if $\langle q\rangle' \neq 0$ everywhere on $[-a, a]$, $\gamma_k = 0$ necessarily, implying stability. For $\gamma_k \neq 0$ (i.e. instability),

$$\int_{-a}^{a} dy \frac{k_x\partial_y\langle q\rangle}{(\omega_r - k_x V_x(y))^2 + \gamma_k^2}|\tilde{\phi}_k|^2 = 0 \tag{4}$$

is required. This implies that $\langle q\rangle'$ *must* change sign at some point on the interval $[-a, a]$, i.e. for example, $\langle q\rangle' < 0$ on $[-a, x]$ and $\langle q\rangle' > 0$ on $[x, a]$. Since $\langle q\rangle' = \partial_y\langle\partial_y^2\phi\rangle = \partial_y^2\langle V_x(y)\rangle$,

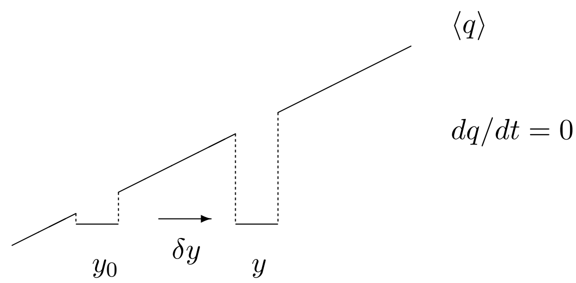

FIG. 6. Displacement of PV blob

the slope of the vorticity must change somewhere in $[-a, a]$, or equivalently the mean flow profile must have an *inflection point* somewhere on $[-a, a]$. We have arrived at the essence of the famous *Rayleigh inflection point theorem*, namely that a necessary condition for inviscid instability of a shear flow is that the flow must have an inflection point. Fjortoft later showed that, for instability, the inflection point must be a vorticity maximum[2, 4].

B. Something Newer: G.I. Taylor's Stability Criterion Derived from PV Dynamics

The Rayleigh inflection point theorem is based entirely on manipulations of the equation and so is not very satisfying from a physics perspective. More substantively, Rayleigh's criterion does not address the possibility that excitons or structures other than linear eigenmodes (i.e. waves) may be the optimal ones with which to tap the available free energy in the shear. This brings us to G.I. Taylor's argument from his famous paper of 1915[5]. Taylor constructed a very physical and intuitive description of the consequence of infinitesimal displacement of a 'blob' or 'slug' of potential vorticity (PV) in a system which locally conserves PV. The class of such systems includes non-dissipative 2D fluids, quasi-geostrophic fluids, and both Hasegawa-Mima and Hasegawa-Wakatani system plasmas. The crux of Taylor's argument is on the analysis of consequences for the mean flow when a 'blob' of PV is displaced up or down the mean cross-stream PV gradient, as shown in Fig. 6.

To address the effect on the flow, consider the momentum balance equation for inviscid, isentropic displacements

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = -\nabla h \tag{5a}$$

where h is enthalpy (i.e. $dh = dp/\rho$). Equivalently,

$$\frac{\partial \mathbf{V}}{\partial t} = -\nabla \left(h + \frac{V^2}{2} \right) + \mathbf{V} \times \boldsymbol{\omega} \tag{5b}$$

So, for the mean component,

$$\frac{\partial \langle \mathbf{V} \rangle}{\partial t} = \langle \mathbf{V} \times \boldsymbol{\omega} \rangle \tag{5c}$$

where we assumed symmetry in $\hat{x}$ direction. For the zonal component:

$$\frac{\partial \langle V_x \rangle}{\partial t} = \langle \tilde{V}_y \tilde{\omega}_z \rangle \tag{6a}$$

or equivalently

$$\frac{\partial \langle V_x \rangle}{\partial t} = \langle \tilde{V}_y \tilde{q} \rangle \tag{6b}$$

since $\tilde{q} = \tilde{\omega}_z$ for $q = \omega_z + \beta y$, as for quasigeostrophic (QG) fluids. Note that since $\tilde{\omega}_z = -(\partial_x^2 + \partial_y^2)\tilde{\phi}$, performing the zonal average gives

$$\frac{\partial \langle V_x \rangle}{\partial t} = -\partial_y \langle \tilde{V}_y \tilde{V}_x \rangle \tag{6c}$$

Eqs. (6a), (6b), and (6c) state the famous, and even useful, *Taylor Identity.*

$$\langle \tilde{V}_y \tilde{\omega}_z \rangle = \langle \tilde{V}_y \tilde{q} \rangle = -\partial_y \langle \tilde{V}_y \tilde{V}_x \rangle \tag{6d}$$

The Taylor Identity states that for a 2D or QG, etc fluid with conserved vorticity or PV, the cross zonal stream flux of PV equals the along-stream component of the Reynolds force. Taylor's identity establishes that vorticity transport is the process which controls the dynamics of zonal flows and their self-acceleration.

Returning to the issue of stability, we note that to utilize Eq.(6b), we must calculate $\tilde{q}$. Now

$$\tilde{q} = (\text{PV of vortex blob at } y) - (\text{Mean PV at } y) \tag{7a}$$

Since $\tilde{q}(y)$ is displaced to y from y_0, and since $dq/dt = 0$ with initial fluctuations negligibly small, we have

$$(\text{PV of vortex blob at } y) = \langle q(y_0) \rangle \tag{7b}$$

Also, for small $y - y_0 \equiv \delta y$, we can 'Taylor' expand:

$$\langle q(y)\rangle \cong \langle q(y_0)\rangle + (y - y_0)\frac{d\langle q\rangle}{dy}\bigg|_{y_0} \tag{7c}$$

So

$$\tilde{q} \cong -\delta y\, \frac{d\langle q\rangle}{dy}\bigg|_{y_0} \tag{7d}$$

Since the choice of y_0 is arbitrary, we hereafter drop the subscript. Thus, we finally arrive at

$$\frac{\partial\langle V_x\rangle}{\partial t} = -\langle\tilde{V}_y\delta y\rangle\frac{d\langle q\rangle}{dy} \tag{8a}$$

or, taking the fluid parcel displacement $\tilde{\xi} = \delta y$ and noting $\partial_t\tilde{\boldsymbol{\xi}} = \mathbf{V}$,

$$\frac{\partial\langle V_x\rangle}{\partial t} = -\left(\partial_t\frac{\langle\tilde{\xi}^2\rangle}{2}\right)\frac{d\langle q\rangle}{dy} \tag{8b}$$

Instability of the system to the initial PV slug displacement requires, of course, $\partial_t\langle\tilde{\xi}^2\rangle > 0$. At the same time, the net momentum of the flow is conserved, so

$$\frac{\partial}{\partial t}\int_{-a}^{a} dy\langle V_x\rangle = -\int_{-a}^{a} dy\left(\partial_t\frac{\langle\tilde{\xi}^2\rangle}{2}\right)\frac{d\langle q\rangle}{dy} = 0 \tag{9}$$

Since $\partial_t\langle\tilde{\xi}^2\rangle > 0$, total momentum conservation requires that $\partial\langle q\rangle/\partial y$ *must* change the sign at some point on the interval $[-a, a]$. For $q = \nabla^2\phi$, this is equivalent to requiring that the flow has an inflection point.

Taylor's derivation of the Rayleigh result is notable in that:

1. it makes no reference to waves or eigenmodes, but rather is formulated in terms of the displacement of a 'blob' or 'slug' of vorticity. Of course, it is limited to consideration of a small displacement.

2. it directly links stability to flow evolution, and so is useful for obtaining more general insights into dynamics.

The physical clarity and simplicity of Taylor's derivation make it far more satisfying than Rayleigh's. We should mention here that there were at least two notable follow-ons to Taylor's analysis. First, C.C. Lin[6] showed that a flow profile inflection point is needed to allow the interchange of two vortices without a restoring force, so that the shear layer relaxes. Lin's analysis was based on using the Biot-Savart law to calculate the force on individual vortex elements. Later, V.I. Arnold[7] presented a famous *nonlinear* stability analysis which showed that the existence of a flow profile with an inflection point is still a necessary condition for instability.

C. Something Further: Zonal Flow Evolution and Wave-Flow Interaction

Recall that Eq.(8b) states that

$$\frac{\partial \langle V_x \rangle}{\partial t} = -\left(\partial_t \frac{\langle \tilde{\xi}^2 \rangle}{2}\right) \frac{d\langle q \rangle}{dy} \tag{10}$$

and so relates the mean zonal flow evolution to the mean PV gradient and the evolution of the displacement of a fluid element[8]. Note that since the vortex element or PV diffusivity D_q is, by definition,

$$D_q = \partial_t \frac{\langle \tilde{\xi}^2 \rangle}{2} \tag{11a}$$

it follows that

$$\frac{\partial \langle V_x \rangle}{\partial t} = -D_q \frac{d\langle q \rangle}{dy} \tag{11b}$$

Eq. (11b) is an important, general result which states that *a latitudinal diffusive flux of potential vorticity will accelerate a zonal flow.* This is in accord with the general concept that *links zonal flow formation to PV mixing.* Indeed, recently M. McIntyre and R.Wood[9] published a lengthy discussion of this issue which generalizes, but draws heavily upon, Taylor's pioneering insights. Eq. (11b) prescribes the direction of the zonal acceleration. For $d\langle q \rangle/dy > 0$, the acceleration is westward, while for $d\langle q \rangle/dy < 0$, the acceleration is eastward. In particular, this suggests that the beta effect will drive a westward circulation ($\beta > 0$). In general, any PV mixing process which tends to increase the variance of the latitude of a fluid particle (i.e. $\partial_t \langle \tilde{\xi}^2 \rangle > 0$), will accelerate a zonal flow opposite to $d\langle q \rangle/dy$. Finally, we observe that Eq. (11b) can also be obtained using the Taylor Identity (Eq. (6d)) and a mean field calculation (i.e. quasilinear theory) for the vorticity flux. We leave this as an exercise for the readers.

Taylor's argument for zonal flow generation is much more fundamental and elegant than modulational instability methods and the other cranks we love to turn[10]. The only two essential elements in his argument are local PV conservation along fluid trajectories and PV mixing, i.e. the net irreversible transport of potential vorticity. This begs the question of *what is the origin of irreversibility in the PV flux*? Equivalently, what is the microscopic mechanism for PV mixing? In the language of MFE methodology, this boils down to calculating the cross-phase in the PV flux $\langle \tilde{V}_y \tilde{q} \rangle$. There are several viable candidates, which include:

1. direct dissipation, as by viscosity

2. nonlinear coupling to small scale dissipation by the *forward* cascade of potential vorticity.

3. Rossby or drift wave absorption at critical layers, where $\omega = k_x \langle V_x(y) \rangle$. This is essentially Landau resonance. Transport or mixing of PV region requires the overlap of neighboring critical layers, leading to stochastization of flow streamlines.

4. stochastic nonlinear wave-fluid element scattering, which is analogous to transport induced by nonlinear Landau damping.

Note that the more general concepts are stochasticity of streamlines and forward potential enstrophy cascade to small scale dissipation. Interestingly, when looking at the phenomenon of zonal flow self-organization from the standpoint of PV transport and mixing, it is the *forward enstrophy cascade* which is critical, and not the inverse energy cascade, as is conventionally mentioned!! Finally, we observe that zonal flow acceleration is not necessarily a strongly nonlinear process. PV mixing can occur via wave absorption, and can be manifested in weak turbulence, as an essentially quasilinear process. In this regard, the reader should consult [11].

Eqs. (10) and (11b) also lead to a useful and instructive momentum conservation theorem for wave-mean flow interaction. From these, we immediately see that

$$\frac{\partial}{\partial t}\left\{\langle V_x\rangle + \frac{\langle\tilde{\xi}^2\rangle}{2}\frac{\partial\langle q\rangle}{\partial y}\right\} = 0 \tag{12a}$$

Using the relation

$$\tilde{q} = -\tilde{\xi}\frac{\partial\langle q\rangle}{\partial y} \tag{12b}$$

which relates PV perturbation to fluid element displacement, we can then re-write Eq. (12a) as

$$\frac{\partial}{\partial t}\left\{\langle V_x\rangle + \frac{\langle\tilde{q}^2\rangle}{2(\partial\langle q\rangle/\partial y)}\right\} = 0 \tag{12c}$$

It is interesting then to note that for $\partial\langle q\rangle/\partial y = \beta$ and $\tilde{q} = \nabla^2\tilde{\phi}$, we find from Eq. (12c)

$$\frac{\partial}{\partial t}\left\{\langle V_x\rangle - \sum_{\mathbf{k}} k_x N_{\mathbf{k}}\right\} = 0 \tag{12d}$$

where

$$N_{\mathbf{k}} = \frac{E(k)}{\omega_{\mathbf{k}}} \tag{12e}$$

is the wave action density for Rossby waves. $E(k)$ is simply the Rossby wave energy density. We see, then that Eq. (12d) is a momentum theorem which ties the zonal flow momentum $\langle V_x\rangle$ to the zonal wave momentum density $k_x N_{\mathbf{k}}$, which is the negative of the pseudomomentum. Eq. (12d) states that the zonal flow cannot 'slip' relative to the wave momentum density, or equivalently, that the quasi-particle field of the wave packets is frozen into the zonal flow. Eq. (12d) is a limiting case of the Charney-Drazin non-acceleration theorem, which states that in the absence of sources and sinks, the zonal flow momentum can change *only* if the wave momentum density varies in time. The full Charney-Drazin theorem[4, 12, 13] for the QG equation is:

$$\frac{\partial}{\partial t}\left\{\langle V_x\rangle + \frac{\langle\tilde{q}^2\rangle}{2(\partial\langle q\rangle/\partial y)}\right\} = -\nu\langle V_x\rangle + \frac{1}{\langle q\rangle'}\left\{\langle\tilde{f}^2\rangle\tau_c - \mu\langle(\nabla\tilde{q})^2\rangle - \partial_y\langle\tilde{V}_y\tilde{q}^2\rangle\right\} \tag{13}$$

Here $\tilde{f}$ is the stochastic forcing which drives the system, τ_c is its correlation time, μ is the viscosity, and ν is the scale invariant drag. The term $\partial_y\langle\tilde{V}_y\tilde{q}^2\rangle$ accounts for the local convergences and divergences in the flux of potential enstrophy - i.e. turbulence spreading[14]. Physically speaking, Eq. (13) states that the sum of the flow momentum and the wave pseudomomentum is conserved up to frictional flow damping and turbulence excitation ($\sim \langle \tilde{f}^2 \rangle \tau_c$), dissipation ($\sim \mu \langle (\nabla \tilde{q})^2 \rangle$) and spreading. Thus, stationary turbulence can drive a zonal flow *only* by forcing, dissipation or convergence of spreading, all of which break the local freezing-in law for quasi-particle momentum and flow momentum. For stationary flow and turbulence, we find

$$\langle V_x \rangle = \frac{1}{\nu \langle q \rangle'} \left\{ \langle \tilde{f}^2 \rangle \tau_c - \mu \langle (\nabla \tilde{q})^2 \rangle - \partial_r \langle \tilde{V}_r \tilde{q}^2 \rangle \right\} \tag{14}$$

Thus, the flow direction is ultimately set by $\langle q \rangle'$ and the spatial distribution of sources and sinks. Note that some finite stand-off distance between source and sink is required for a finite scale shear flow. In simple drift wave turbulence, this is usually provided by the disparity and distance between the electron coupling region of the wave and the ion Landau resonance point, where wave absorption occurs.

Formally, the Charney-Drazin theorem is a consequence of, and is derived from, potential enstrophy density balance. The pseudomomentum density is simply the fluctuation potential enstrophy density divided by $\langle q \rangle'$. Thus, we see that a physical perspective on the pseudomomentum is that the fluctuation potential enstrophy density sets its size, while $\langle q \rangle'$ defines its direction or orientation. Alternatively, note that the pseudomomentum density is a measure of the effective 'roton' intensity field. In this vein, it is easy to see why potential enstrophy flux convergence ($\sim \partial_r \langle \tilde{V}_r \tilde{q}^2 \rangle$) affects the zonal momentum balance, since it alters the roton quasi-particle density. Interestingly, drift waves have the character of *both* types of quasiparticles: namely phonons, via the relation $N_{\mathbf{k}} = E(k)/\omega_{\mathbf{k}}$ and rotons, via the proportionality of pseudomomentum to $\langle \tilde{q}^2 \rangle / \langle q \rangle'$[15]. Ultimately, this dual character is a consequence of the fact that both energy and potential enstrophy are inviscid invariants of the system.

III. DYNAMICS OF PHASE SPACE STRUCTURES

A. Basic Concepts

In this section, we discuss phase space structures in Vlasov turbulence[16–18]. That there should be a close correspondence between PV dynamics in QG and drift wave turbulence on one hand, and Vlasov turbulence on the other hand, is no surprise, since both systems are governed by the incompressible advection of a conserved quantity along Hamiltonian trajectories, with feedback via a Poisson equation. Table I compares the two systems and is self-explanatory. One point which should be emphasized is that in addition to Hamiltonian advection of a conserved scalar field, both QG and Vlasov turbulence satisfy a Kelvin's theorem and thus have a conserved circulation. We remark that Donald Lynden-Bell was the first to prove a Kelvin's theorem for the Vlasov equation as an appendix to his pioneering paper[19] on violent relaxation in 1967. These are fundamental to several of the common features of their dynamics. In particular, the conservation of q, the total PV density and of f, the total phase space density, respectively leads to interesting relations concerning the evolution of a localized blob or structure, as well as its interaction with the mean gradient, $d\langle q\rangle/dy$ or $\partial\langle f\rangle/\partial v$. For PV, we have ($\langle q\rangle$ is the mean distribution)

$$\frac{d}{dt}(\langle q\rangle + \delta q) = 0 \tag{15a}$$

So

$$\frac{\partial}{\partial t}\int d^2x\delta q^2 = -2\frac{d}{dt}\int d^2x\langle q\rangle\delta q \tag{15b}$$

Expanding $\langle q\rangle$ around a localized point y_0, we have

$$\langle q\rangle = \langle q(y_0)\rangle + (y - y_0)\left.\frac{d\langle q\rangle}{dy}\right|_{y_0} \equiv q_0 + \delta y\frac{dq_0}{dy_0} \tag{15c}$$

where q_0 is a static, initial PV distribution. Then Eq.(15b) gives

$$\frac{\partial}{\partial t}\int d^2x\delta q^2 = -2\frac{dq_0}{dy_0}\int d^2x\langle\tilde{V}_y\delta q\rangle \tag{15d}$$

The reader can easily see that after utilizing the Taylor Identity (Eq.(6d)) and the mean flow evolution equation with scale independent drag, we recover a limit of the Charney-Drazin

	QG system	Vlasov system
Basic Equation	$\partial_t q + \{q, \phi\} - \nu\nabla^2 q = 0$	$\partial_t f + \{f, H\} = C(f)$
Field	PV, $q = \beta y + \nabla^2\phi$ $q = \ln n_0(r) + \phi - \rho_s^2\nabla^2\phi$	distribution function $f(x, v, t)$
Evolution	$\{q, \phi\}$	$\{f, H\}$
Dissipation/Coarse Graining	$-\nu\nabla^2$	$C(f)$
Poisson Eq./Feedback	$q = q_0 + \beta y + \nabla^2\phi$	$\nabla^2\phi = -4\pi n_0 q \int f dv$
Kelvins' Theorem	$\oint(V + 2\Omega a \sin\theta)dl = const$	$\oint \mathbf{v}(s) \cdot d\mathbf{x} = const$
Decomposition	Planetary + Relative $\Rightarrow q = \beta y + \omega$	$f = \langle f\rangle + \tilde{f}$
Physical Element	vortex	granulation $\rightarrow$ hole, clump
Conservation	$\int q da$ (i.e. total charge)	phase volume

TABLE I. Comparison of PV and Vlasov system

theorem. Note that Eq.(15d) can be re-written as:

$$\frac{\partial}{\partial t}\int d^2x\delta q^2 = 2\left(\frac{dq_0}{dy}\right)^2 \int d^2x \frac{d}{dt}\frac{\langle\delta y^2\rangle}{2} \tag{15e}$$

For the corresponding case of a Vlasov plasma structure[17], we have:

$$f = \langle f\rangle + \delta f \tag{16a}$$

So

$$\frac{\partial}{\partial t}\int dv\delta f^2 = -2\frac{d}{dt}\int dv\langle f\rangle\delta f \tag{16b}$$

Again expanding,

$$\langle f\rangle = f_0 + (v-u)\left.\frac{\partial f_0}{\partial v}\right|_u = f_0 + (v-u)\frac{\partial f_0}{\partial u} \tag{16c}$$

we find

$$\frac{\partial}{\partial t}\int dv\delta f^2 = -2\left(\frac{\partial f_0}{\partial u}\right)\frac{d}{dt}\int dv(v-u)\delta f \equiv -2\left(\frac{\partial f_0}{\partial u}\right)\frac{d}{dt}\frac{\langle p\rangle}{m} \tag{16d}$$

Here $\langle p \rangle$ is the blob averaged momentum. In deriving Eq.(16d), we have assumed that the characteristic oscillation frequency for an individual particle in the localized phase space structure (i.e. "bounce frequency") is high compared to the structure growth rate and the mean relaxation rate, i.e. $\omega_b \gg (\partial_t \delta f)/\delta f, (\partial_t \langle f \rangle)/\langle f \rangle$. Eq.(16d) is remarkable in that it links local structure growth to the evolution of net, *local* momentum. Depending upon the signs of $\partial f|_0/\partial u$ and the possibilities for $d\langle p \rangle/dt$, structure *growth* or *damping* is possible. The key to this is the nature of possible momentum exchange channels.

B. Local Structure Growth

In this section, we discuss the dynamics of nonlinear growth of localized Vlasov structures. Before launching into terra nova, we revisit terra firma, so as to place new ideas in the familiar (i.e. boring) context of the same old same old. A staple of Vlasov microstability theory is the current driven ion-acoustic (CDIA) instability[18, 20], which is also important for anomalous resistivity. As shown in Fig.7, CDIA is essentially a battle between inverse electron resonance and ion resonance. The shift in the electron mean drift velocity v_d must satisfy $v_d > c_s$, so $\omega - kv_d < 0$. For

$$\epsilon = \epsilon_r + i\epsilon_{IM} \tag{17a}$$

and

$$\gamma_{\mathbf{k}} = -\frac{\epsilon_{IM}}{(\partial\epsilon_r/\partial\omega)|_{\omega_{\mathbf{k}}}} = -\frac{\epsilon^e_{IM} + \epsilon^i_{IM}}{(\partial\epsilon_r/\partial\omega)|_{\omega_{\mathbf{k}}}} \tag{17b}$$

the algebra confirms the picture of Fig.7, namely that the shift v_d must be large enough so that inverse electron Landau damping exceeds the positive ion Landau damping. As we all learned in kindergarten, this is most easily realized for *minimal* overlap of the ion and electron distribution functions. From the perspective of anomalous resistivity, the requisite momentum exchange is between electrons and *waves*, as opposed to collisional resistivity in which electrons exchange momentum with *ions*, by particle collisions. CDIA saturation requires some sort of *nonlinear* dissipation process, including possibly nonlinear ion Landau damping, in order to dispose of the wave energy.

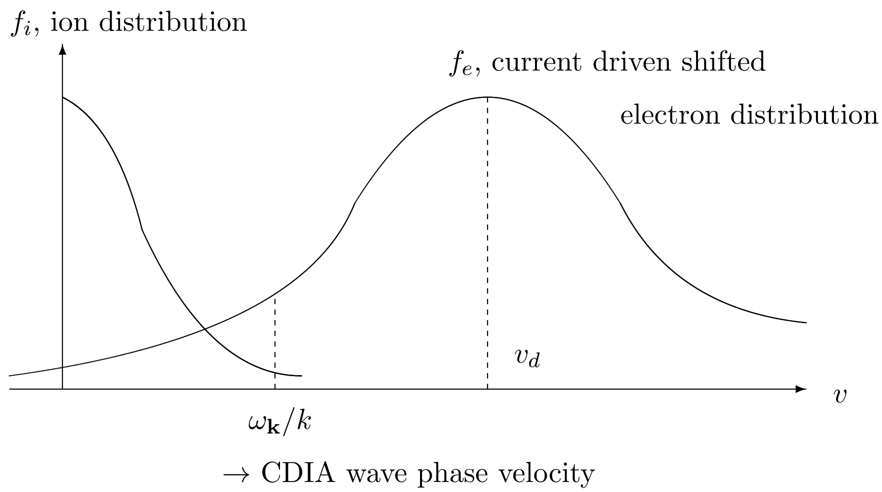

FIG. 7. CDIA

The situation for localized structures is different, since several possible channels for momentum exchange exist. In general, momentum conservation implies

$$\frac{d}{dt}(\langle p_i\rangle + p_e + p_w) = 0 \tag{18}$$

where we assumed that our blob was an *ion* structure (this choice is arbitrary!) and p_e, p_w correspond to electron and wave momentum, respectively. Thus in a one species plasma (dynamically), with u off wave resonance, $d\langle p_i\rangle/dt = 0$ so structure growth is impossible. However, in a two species plasma more interesting things can happen. In particular, for ions:

$$\frac{d}{dt}\langle p_i\rangle = -\frac{1}{2}\frac{m_i}{(\partial f_{0,i}/\partial v)|_u}\partial_t \int dv\delta f_i^2 \tag{19a}$$

and for electrons

$$\frac{d}{dt}p_e = -\frac{1}{2}\frac{m_e}{(\partial f_{0,e}/\partial v)|_u}\partial_t \int dv\delta f_e^2 \tag{19b}$$

So momentum conservation (with $\partial p_w/\partial t \cong 0$) requires

$$\frac{m_e}{(\partial f_{0,e}/\partial v)|_u}\partial_t \int dv\delta f_e^2 = -\frac{m_i}{(\partial f_{0,i}/\partial v)|_u}\partial_t \int dv\delta f_i^2 \tag{19c}$$

Thus, we see that if:

$$\left.\frac{\partial f_{i,0}}{\partial v}\right|_u \left.\frac{\partial f_{e,0}}{\partial v}\right|_u < 0 \tag{19d}$$

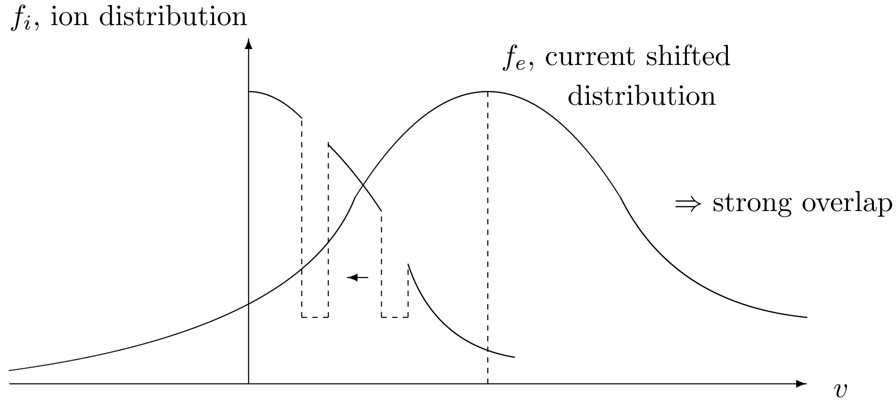

FIG. 8. Ion hole growth in current carrying Vlasov plasma

- i.e. the two mean distributions have *opposite* slopes at the structure velocity - growth of δf is possible[17]. This is essentially a statement of availability of free energy - i.e. the presence of a finite v_{de}. Here, growth of localized structures occurs by collisionless inter-species momentum exchange, which move the structure up (for hole) or down (for blob) the mean phase space density gradient. Conservation of phase space density then requires that the fluctuation amplitude grows. See Fig.8 for a demonstration. It is interesting to contrast the conditions for structure growth with the familiar conditions for CDIA wave growth. Structure growth requires *strong* overlap of electron and ion distributions, with opposite slope. Overlap of $\langle f \rangle$ facilitates inter-species momentum exchange, which can propel a structure up or down the gradient. CDIA, by contrast, requires minimal overlap for instability. Minimal overlap reduces the stabilizing effects of ion Landau damping. Also, we note that Eq.(19a) effectively states $\partial p/\partial t \sim \partial_t \int dv \delta f^2/(\partial f_0/\partial v)|_u$, so we see the appearance of $\int dv \delta f^2/(\partial f_0/\partial v)|_u$ as a pseudomomentum. Here, however, we are not necessarily speaking of *linear waves*, but of more general types of fluctuations. Hence this observation attests to the fact that the pseudomomentum is a more general concept. It is comforting, however, to note that plugging the non-resonant linear response in for δf recovers the conventional expression for wave momentum density as derived from small amplitude theory[21].

There is an old injunction to lecturers which advises: 'Don't try to prove everything, but do try to prove at least one thing'. Thus, we now try to calculate the growth rate of an ion phase space structure (hole) driven by an electron current. From Eq.(19a), we have

$$\frac{1}{2}\frac{m_i}{(\partial f_{0,i}/\partial v)|_u}\partial_t \int dv \delta f_i^2 = -\frac{d\langle p_i\rangle}{dt} = \frac{dp_e}{dt} \tag{20}$$

Now, while ion orbits are trapped, forming a phase space vortex, we can hope to claculate the orbits of the electrons (with much smaller inertia!) by assuming weak deflection and using mean field theory. It is a standard crank to then show

$$\frac{\partial\langle f_e\rangle}{\partial t} = \frac{\partial}{\partial v}D(v)\frac{\partial}{\partial v}\langle f_e\rangle \tag{21a}$$

where

$$D(v) = \frac{q^2}{m_e^2}\sum_{k\omega}\langle E^2\rangle_{k\omega}\pi\delta(\omega - kv) \tag{21b}$$

is the usual quasilinear diffusion coefficient (but note $\omega \neq \omega(k)$ here!). Then,

$$\begin{aligned} \frac{dp_e}{dt} &= -m_e \int dv D(v)\frac{\partial}{\partial v}\langle f_e\rangle \\ &\simeq -m_e D(u) \left.\frac{\partial f_{0,e}}{\partial v}\right|_u \Delta u \end{aligned} \tag{21c}$$

where we have assumed the scattering spectrum to have a phase velocity distribution peaked near u. Δu is the approximate width of the distribution. Then combining Eqs. (20) and (21c) gives:

$$\partial_t \int dv\delta f_i^2 \cong -\frac{m_e}{m_i}\left.\frac{\partial f_{0,i}}{\partial v}\right|_u \int dv D(v)\frac{\partial}{\partial v}\langle f_e\rangle \tag{21d}$$

$$\cong -\frac{m_e}{m_i}\Delta u D(u) \left.\frac{\partial f_{0,i}}{\partial v}\right|_u \left.\frac{\partial f_{0,e}}{\partial v}\right|_u \tag{21e}$$

Once again, we see that the need for strong electron and ion distribution function overlap at a location of opposite slope - the condition for available free energy. To extract the key scalings of the growth rate, we note from Eq.(21b) that $D(v) \sim \langle\tilde{E}^2\rangle \sim \langle\delta f^2\rangle(\Delta v_T)^2$. Here Δv_T is extent of the phase space structure in velocity. Loosely speaking, it corresponds to a self-trapping width, i.e. the width in velocity of a bunch of resonant particles which define a structure[17]. In resonance broadening theory[22], for a 1D plasma, we have:

1. the spectral auto-correlation time

$$\tau_{ac} = \left[\Delta k\left(\frac{d\omega}{dk} - \frac{\omega}{k}\right)\right]^{-1}$$

which defines the time of self-coherence of a wave packet

2. the wave particle correlation time

$$\tau_c = (k^2 D)^{-1/3}$$

which defines the time it takes for a particle to scatter one wavelength, i.e. to decorrelate from the wave by random kicks in velocity

3. the $\langle f \rangle$ relaxation time, τ_{relax}

We always assume the ordering $\tau_{ac} < \tau_c < \tau_{relax}$. In the vein, then, Δv_T is defined by

$$\Delta v_T = \frac{1}{k\tau_c} \tag{22}$$

Of course, $\int dv \delta f^2 \sim \Delta v_T \delta f^2$, so the growth rate has the form

$$\gamma \simeq k\Delta v_T \left[-\frac{\partial f_{0,i}}{\partial v} \frac{\partial f_{0,e}}{\partial v} \right]_u F(\text{mess}) \tag{23}$$

Here $F(\text{mess})$ is a complicated function related to the details of the phase space structure, and is of no instructive value. Note that the growth rate is nonlinear, i.e. $\gamma \sim k\Delta v_T \sim \omega_b \sim (q\phi/m\Delta x)^{1/2}$. Here ϕ is the self-potential of the phase space structure, as determined by Poisson's equation, and Δx is the spatial extent of the structure. Note we tacitly define a 'structure' as a blob/hole perturbation of size δf, spatial extent Δx and extent in velocity Δv_T. Obviously, $\tilde{\phi}_{self} \sim \delta f \Delta v_T / \epsilon(k, kv)$ where $\epsilon(k, kv)$ is the dielectric function evaluated at the ballistic frequency of the structure centroid motion. Eq.(23) suggests that structure growth will continue until the free energy source is depleted. Thus, we can expect an ion hole to be accelerated up the velocity profile till $(\partial f_{0i}/\partial v)(\partial f_{0,e}/\partial v) \to 0$.

As mentioned previously, $\int dv[\delta f^2/(\partial f_0/\partial v)]$ constitutes a pseudomomentum or effective dynamical pressure for the structure. From the Vlasov equation, we can immediately show that

$$\partial_t \int dv \frac{\langle \delta f_i^2 \rangle}{\partial f_0/\partial v|_u} = -\frac{q}{m_i} \langle \tilde{E} \delta n_i \rangle \tag{24a}$$

The RHS is, of course, simply the force on the ions, so we thus arrive at a 'Charney-Drazin' theorem for Vlasov turbulence

$$\frac{d}{dt}\left\{\int dv \frac{\langle \delta f_i^2 \rangle}{\partial f_0/\partial v|_u} + \frac{\langle p_i \rangle}{m_i}\right\} = 0 \tag{24b}$$

We can trivially derive a corresponding relation for electrons, and then use momentum balance to re-derive Eq.(24b). The point of this observation is to illustrate the close correspondence between the Charney-Drazin theorem and the theory of nonlinear phase space structure growth. Both are grounded in concepts of fluctuation pseudomomentum and the conservation of an effective phase space density (q or f) along Hamiltonian trajectories. Thus, it is not surprising that one should reduce to the other.

Another topic worthy of mention at this stage is the relation of phase space structure dynamics to the Berk-Breizman (B-B) model[23–26]. The B-B model is a 1D model, including resonant particles, waves (with damping), and collisions. It represents the present day extension of classic linear bump-on-tail problem[27, 28], the Laval-Pesme extension thereof[29], and the traveling wave tube system studied experimentally by Malmberg, et. al.[30]. Constructed to mimic the essential features of resonant particles in Toroidal Alfven Eigenmodes in a simpler, more tractable context, the B-B hides a wealth of physics in the 'plain paper wrapping' of a simple model. In particular trapping, structure formation and cyclic bursts are all possible. The basic equations of the B-B model are:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + E\frac{\partial f}{\partial v} = C(f - f_0), \tag{25a}$$

the collision operator

$$C(f - f_0) = \frac{\nu_f^2}{k}\frac{\partial}{\partial v}(f - f_0) + \frac{\gamma_d^3}{k^2}\frac{\partial^2}{\partial v^2}(f - f_0), \tag{25b}$$

and the effective Poisson equation (really displacement current relation)

$$\frac{\partial E}{\partial t} = -\int dvv(f - f_0) - 2\gamma_d E. \tag{25c}$$

Here γ_d is the external damping associated with all 'other' processes, including wave damping, second species, etc. Collisions can be important here, as a means to limit plateau persistence in time and to de-trap particles.

Ignoring collisions, an analysis like that given above yields

$$\partial_t \int dv \langle \delta f^2 \rangle = - \left. \frac{\partial f_0}{\partial v} \right|_u \int dv \langle \tilde{E} \delta f \rangle \tag{26a}$$

where we take δf located at speed u localized in velocity relative to $\langle f \rangle$, i.e. $\Delta v < v_{Th}$. Then, Eq.(25c) gives

$$(-i\omega + 2\gamma_d) E_{k,\omega} = - \int dv v \delta f \tag{26b}$$

So

$$\partial_t \int dv \langle \delta f^2 \rangle = 2\gamma_d \left. \frac{\partial f_0}{\partial v} \right|_u \int dv' \int dv \frac{v' \langle \delta f(v') \delta f(v) \rangle}{(ku)^2 + (2\gamma_d)^2} \tag{26c}$$

Taking $dv \sim \Delta v$, $v' \sim u$, $\omega \sim ku$ etc. then gives

$$\gamma \cong (\Delta v) \left. \frac{\partial f_0}{\partial v} \right|_u \frac{2\gamma_d u}{(ku)^2 + (2\gamma_d)^2} \tag{26d}$$

Of course, we see there is no free lunch - i.e. there must be free energy, so $(\partial f_0 / \partial v)|_u > 0$ is required for instability. However, we do note that:

1. the growth is *nonlinear*, i.e. $\gamma \sim \Delta v$

2. *linear* instability is *not* required, i.e. γ can be positive here even if $\gamma_{L,0} - \gamma_d < 0$, where $\gamma_{L,0} = \pi (\partial f_0 / \partial v)|_{\omega/k} / 2k^2$ is the usual bump-on-tail drive rate.

We also note that the sign of δf and the relation between δf and Δv must be determined by an analysis of the condition for Jeans equilibrium. We remark here that it would be interesting to explore the effects of collisions on the B-B nonlinear structure growth process.

IV. PHASE SPACE STRUCTURE DYNAMICS IN DRIFT TURBULENCE

In this section, we extend our discussion of phase space structure dynamics from the case of 1D to the more relevant problem of drift turbulence. This is a challenging, vast and still developing subject. Thus, we limit our treatment to: a.) the presentation and discussion of the Darmet model, a useful prototype which is both simple and relevant, b.) a discussion of the dynamics of zonal flow formation induced by relaxation of a localized structure and the relation of this process to flow momentum plus pseudomomentum conservation, c.) a calculation of drift structure growth, including both spatial and velocity scattering.

A. Introducing the Darmet Model

The Darmet model[31], which is really an extended version of the Tagger-Pellat[32]-Diamond-Biglari[33] model descibes the dynamics of $f(x, y, E, t)$, which corresponds to a bounce averaged trapped ion distribution function. The basic equations are

$$\partial_t f + v_d \partial_y f + \{f, \phi\} = C(f) \tag{27a}$$

$$\alpha_e(\phi - \langle\phi\rangle_y) - \rho^2\nabla^2\phi = \frac{2}{n_{eq}\sqrt{\pi}}\int_0^\infty dE\sqrt{E}f - 1 \tag{27b}$$

with heat flux Q matched according to:

$$Q = -\chi_{coll}\langle T\rangle' + \int dE\sqrt{E}E\langle\tilde{V}_r\delta f\rangle \tag{27c}$$

given by the sum of turbulent and neoclassical processes. Note that $v_d = v_{d,0}(E/E_{th})$ is an energy dependent precession drift velocity. $\rho^2\nabla^2$ accounts for polarization charge, due to both FLR and finite banana width. The linear waves manifested by the Darmet model are trapped ion ITG modes, and the model is easily extended to include non-Boltzmann electron response. Dissipative trapped electron dynamics are of particular relevance and simplicity. In the collisionless limit, irreversibility appears here as a consequence of trapped ion precession drift resonance. Note that the constrained nature of the bounce averaged dynamics can force long particle-spectra auto-correlation times, i.e.

$$\begin{aligned}\Delta(\omega - \omega_d) \cong& \Delta k_\theta \left|\frac{d\omega}{dk_\theta} - v_{d,0}\frac{E}{E_{th}}\right| \\ \cong& \Delta k_\theta \left|\frac{d\omega}{dk_\theta} - \frac{\omega}{k_\theta}\right|\end{aligned} \tag{28a}$$

so $\tau_{ac} \sim (|d\omega/dk_\theta - \omega/k_\theta|\Delta k_\theta)^{-1}$ and the Kubo number K is

$$K = \frac{\tilde{v}}{|d\omega/dk_\theta - \omega/k_\theta||\Delta k_\theta|\Delta_r} \tag{28b}$$

Given the weakly dispersive character of long wavelength trapped ion modes, it is very easy for $K \gtrsim 1$ here, even for broad spectra. Thus, quasi-coherent phase space structures are to be expected in the Darmet model.

To form a momentum theorem for the Darmet model, we exploit the connection between polarization charge and fluid vorticity. Balance of $\langle \delta f^2 \rangle$ (akin to enstrophy balance in QG turbulence!) implies

$$\partial_t \langle \delta f^2 \rangle + \partial_r \langle \tilde{v}_r \delta f^2 \rangle - \langle \delta f C(\delta f) \rangle = -\langle \tilde{V}_r \delta f \rangle \langle f \rangle' \tag{29a}$$

so

$$\int dE \frac{\sqrt{E}}{\langle f \rangle'} \left\{ \partial_t \langle \delta f^2 \rangle + \partial_r \langle \tilde{v}_r \delta f^2 \rangle - \langle \delta f C(\delta f) \rangle \right\} = -\langle \tilde{v}_r \delta n_i \rangle \tag{29b}$$

where δn_i is the total ion *guiding center density.* Then the gyrokinetic Poisson equation and our friend the Taylor Identity allow

$$\delta \phi - \rho^2 \nabla^2 \delta \phi = \frac{2}{n_{eq}} \int dE \sqrt{E} \delta f_i \tag{29c}$$

and

$$\langle \tilde{v}_r \delta n_i \rangle = -\langle \tilde{v}_r \rho^2 \nabla \delta \phi \rangle \tag{29d}$$

so, with zonal flow momentum balance

$$\langle \tilde{v}_r \delta n_i \rangle = \partial_t \langle V_\theta \rangle + \nu \langle V_\theta \rangle \tag{29e}$$

we finally arrive at the Charney-Drazin Theorem for zonal flows the Darmet Model:

$$\partial_t \left\{ \mathrm{KPD} + \langle V_\theta \rangle \right\} = -\nu \langle V_\theta \rangle - \int dE \frac{\sqrt{E}}{\langle f \rangle'} \left\{ \partial_r \langle \tilde{v}_r \delta f^2 \rangle - \langle \delta f C(\delta f) \rangle \right\} \tag{29f}$$

Here

$$\mathrm{KPD} = \int dE \sqrt{E} \frac{\langle \delta f^2 \rangle}{\langle f \rangle'} \tag{29g}$$

is the kinetic 'phasetrophy' Density. Of course, $\langle f \rangle' = \partial_r \langle f \rangle$. This strange name is motivated by the obvious resemblance of KPD to potential enstrophy density, which follows from the duality between f in Vlasov turbulence and q in QG turbulence. Given the close connection between pseudomomentum (or wave activity density) and potential enstrophy density in QG turbulence (as discussed in Section II), it is no surprise that KPD corresponds to a kind of kinetic pseudomomentum, formulated directly in terms of δf, with no-apriori linearization or small amplitude assumption. To see this, note that in the small amplitude, non-resonant limit for waves:

$$\delta f_{\mathbf{k}} = -\frac{1}{-i\omega_{\mathbf{k}}}\tilde{v}_{r,k}\langle f\rangle' \tag{30a}$$

so

$$\mathrm{KPD} = \int dE\sqrt{E}\langle\tilde{v}_{r,k}^2\rangle\frac{\langle f\rangle'}{\omega_{\mathbf{k}}^2} \sim -k_\theta\frac{E(k)}{\omega_{\mathbf{k}}} \tag{30b}$$

Thus KPD reduces correctly to pseudomomentum or the negative of the linear wave momentum density in the small amplitude limit. Note that $p_{k,\theta} = k_\theta N_{\mathbf{k}}$, where $N_{\mathbf{k}} = E(k)/\omega_{\mathbf{k}}$ is the wave action density. Eq.(29f) constitutes a non-acceleration theorem for zonal flows in the Darmet Model. In particular, in the absence of KPD evolution, kinetic turbulence spreading or collisions, one cannot accelerate or maintain (against drag) a stationary zonal flow. Alternatively, apart from spreading and collisions, zonal flow growth requires decay of KPD. Thus, we again meet the idea of a law constraining the slippage of a quasi-particle gas relative to the zonal flow.

At this point, the reader - if conscious at all - may be groping for a more physical insight into the nature of KPD. Interestingly, a similar quantity appears in the Antonov Energy Principle[34] for collisionless self-gravitating matter, as discussed in the theory of stellar dynamics[35]. For that energy principle,

$$\delta W = \int d^3x\int d^3v\frac{\delta f^2}{|F_0|'} - G\int d^3xd^3x'd^3vd^3v'\frac{f(\mathbf{x},\mathbf{v})f(\mathbf{x}',\mathbf{v}')}{|\mathbf{x}-\mathbf{x}'|} \tag{31}$$

consists of a fluctuation dynamic pressure term and a self-gravity term. Clearly, the former is the KPD for an unmagnetized plasma. The competition in δW is the usual and familiar one for Jeans instabilities, namely self-gravity vs. dynamical pressure. A similar KPD term appears in the Kruskal-Oberman Energy Principle[36]. Thus, we see that KPD may be thought of as a kinetically founded dynamical pressure.

To conclude this section, it is instructive to compare the Charney-Drazin theorems for Hasegawa-Mima model and Darmet model drift wave turbulence. The relation for H-M turbulence is, from Eq.(13)

$$\frac{\partial}{\partial t}\{\mathrm{WAD} + \langle V_\theta\rangle\} = -\nu\langle V_\theta\rangle - \frac{1}{\langle q\rangle'}\left\{\langle\tilde{f}^2\rangle\tau_c - \mu\langle(\nabla\tilde{q})^2\rangle - \partial_r\langle\tilde{V}_r\tilde{q}^2\rangle\right\} \tag{32a}$$

where WAD $= \langle \tilde{q}^2 \rangle / \langle q \rangle'$, while for the kinetic Darmet model

$$\partial_t \left\{ \mathrm{KPD} + \langle V_\theta \rangle \right\} = -\nu \langle V_\theta \rangle - \int dE \frac{\sqrt{E}}{\langle f \rangle'} \left\{ \partial_r \langle \tilde{v}_r \delta f^2 \rangle - \langle \delta f C(\delta f) \rangle \right\} \tag{32b}$$

where KPD $= \int d^3v \langle \delta f^2 \rangle / \langle f \rangle'$. The correspondence is obvious, thus confirming that conservation of momentum and phase space density really are the key common elements.

B. Phase Space Structures and Zonal Flows

The physical interpretation of KPD becomes problematic for resonant particles. This brings us to the case of a single phase space structure in the Darmet-Model or, more generally, drift wave turbulence. We consider a localized (in phase space!) ion hole or blob, with $\delta f_i = \delta f((x - x_0)/\Delta x,\ (E - E_0)/\Delta E)$, and proceed from phase space density conservation, as before. Thus

$$\frac{df}{dt} = \frac{d}{dt}(f_0 + \delta f) = 0 \tag{33a}$$

so

$$\partial_t \int dE \sqrt{E} \langle \delta f^2 \rangle = -2 \frac{d}{dt} \int dE \sqrt{E} f_0 \delta f \tag{33b}$$

and then

$$f_0 = f_0(x_0) + (x - x_0) \left. \frac{\partial f_0}{\partial x} \right|_{x_0} + ... \tag{33c}$$

We thus have

$$\partial_t \int dE \sqrt{E} \langle \delta f_i^2 \rangle = -2 \langle \tilde{v}_r \delta n_i \rangle \left. \frac{\partial f_0}{\partial x} \right|_{x_0} \tag{33d}$$

where $d(x - x_0)/dt = \tilde{v}_r$, $\int dE \sqrt{E} \delta f_i = \delta n_i$ and assumptions similar to those in Section III apply. Now, a key point enters via the gyrokinetic Poisson equation[37, 38], which relates ion guiding center density δn_i to electron density δn_e and polarization charge density. Thus

$$-\rho^2 \nabla^2 \tilde{\phi} = \delta n_i - \delta n_e \tag{33e}$$

So

$$\partial_t \int dE \sqrt{E} \frac{\langle \delta f_i^2 \rangle}{2} = - \left. \frac{\partial f_0}{\partial x} \right|_{x_0} \left\{ \langle \tilde{v}_r \delta n_e \rangle - \langle \tilde{v}_r \rho_s^2 \nabla^2 \tilde{\phi} \rangle \right\} \tag{33f}$$

The physics of the replacement of Eq.(33d) with Eq.(33f) is a consequence of the fact that the total dipole moment of the plasma is a constant, i.e.

$$\int dx \sum_{\alpha} q_{\alpha} n_{\alpha}(x) x = const \tag{34a}$$

where $n_{\alpha}(x)$ is the density of charge component α. Note that the dipole moment *must* include the polarization component, which in turn guarantees a polarization flux, i.e.

$$-\rho^2 \nabla^2 \phi = \delta n_i(\phi) - \delta n_e(\phi) \tag{34b}$$

So

$$-\langle \tilde{v}_r \rho^2 \nabla^2 \phi \rangle = \langle \tilde{v}_r \delta n_i(\phi) \rangle - \langle \tilde{v}_r \delta n_e(\phi) \rangle \tag{34c}$$

Thus, ambipolarity breaking necessarily implies

$$-\rho^2 \langle \tilde{v}_r \nabla^2 \tilde{\phi} \rangle \neq 0 \tag{34d}$$

Then, using the now all-too-familiar Taylor Identity and flow momentum balance gives

$$\partial_t \left\{ \int dE \sqrt{E} \frac{\delta f_i^2}{2\langle f \rangle'|_{x_0}} + \langle V_\theta \rangle \right\} = -\nu \langle V_\theta \rangle - \langle \tilde{v}_r \delta n_e \rangle \tag{35}$$

which states that even a *localized* phase space blob or hole cannot avoid zonal flow coupling, on account of the fact that spatial scattering of the hole produces a flux of polarization charge due to conservation of total dipole moment.

Eq.(35) can be viewed as a kind of Charney-Drazin theorem for zonal flows produced by localized phase space structure growth. Clearly $\int dE \sqrt{E} \delta f_i^2 / 2\langle f \rangle'$ is the pseudomomentum, and Eq.(35) states that for stationary δf_i, the flow cannot grow unless $-\langle \tilde{V}_r \delta n_e \rangle < 0$ (i.e. $\langle f \rangle' < 0$ is assumed), thus requiring *electron* transport. For, say, dissipative trapped electron response[39] - relevant to trapped ion regime dynamics - we have $\langle \tilde{V}_r \delta n_e \rangle \cong -D_{DT} \partial \langle n \rangle / \partial x$, so

$$\partial_t \left\{ \int dE \sqrt{E} \frac{\delta f_i^2}{2\langle f \rangle'|_{x_0}} + \langle V_\theta \rangle \right\} = -\nu \langle V_\theta \rangle + D_{DT} \frac{\partial \langle n \rangle}{\partial x} \tag{36}$$

where D_{DT} is the dissipative trapped electron diffusivity, $\sim 1/\nu_{e,eff}$. Note that since $\langle f_i \rangle' < 0$ and $\partial \langle n \rangle / \partial x < 0$, Eq.(36) suggests that the electron flux will tend to drive or enhance the ion structure. Physically, this corresponds to ion structure growth by scattering off the (diffusing) electrons, so as to maintain ambipolarity. In the Bump-onTail or CDIA problems, momentum conservation is the key constraint, while for drift wave turbulence, ambipolarity maintenance is central. Note for stationary $\langle \delta f_i^2 \rangle$ and $\langle V_\theta \rangle$ we have

$$\langle V_\theta \rangle = -\frac{\langle \tilde{V}_r \delta n_e \rangle}{\nu} = \frac{D_{DT}}{\nu} \frac{\partial \langle n \rangle}{\partial x} \tag{37}$$

Very clearly:

1. a *localized* structure can excite a *global* (in θ) zonal flow

2. electron transport can drive the nonlinear growth of an ion structure. Note that here, straightforward estimates give $\gamma \sim k v_d \Delta E_T / E_{th}$.

At this point we should remark that it is instructive to compare Eq.(36) to the Charney-Drazin Theorem for the Hasegawa-Wakatani system[13, 40], which is

$$\frac{\partial}{\partial t} \{\mathrm{WAD} + \langle V_\theta \rangle\} = -\nu \langle V_\theta \rangle - \langle \tilde{V}_r \tilde{n} \rangle - \frac{1}{\langle q \rangle'} \left\{ \mu \langle (\nabla \tilde{q})^2 \rangle + \partial_r \langle \tilde{V}_r \tilde{q}^2 \rangle \right\} \tag{38}$$

Here, for H-W system $q = n - \rho_s^2 \nabla^2 \phi$ and $\mathrm{WAD} = \langle \delta q^2 \rangle / \langle q \rangle'$. The correspondence is clear! It is evidently the same equation, allowing for the fact that there is no turbulence spreading or viscous dissipation in the Vlasov-Gyrokinetic structure problem. In particular, for H-W, once again we find that a particle flux can drive the flow against drag. Finally, we note in passing that significant impurity dynamics can have a profound effect on zonal flows by altering the ambipolarity balance.

C. Drift-Hole Growth

In this section, we in essence combine the results of a.) and b.) to derive an expression for drift hole growth. Here a drift hole is a phase space structure in a slab drift wave system. For variety, we consider here an *electron* structure, which is a hole (phase space depression) so as to self-bind. The process of self-binding can be thought of as the formation of a state which is marginally stable to Jeans modes. This is discussed further in references[17, 41]. The growth calculation extends that of ref.[41]. As before, we write

$$\frac{\partial}{\partial t}\int d^3v\delta f^2 = -2\frac{d}{dt}\int d^3v\langle f\rangle\delta f \tag{39a}$$

Now

$$\langle f\rangle = f_0 + (x - x_0)\left.\frac{\partial f_0}{\partial x}\right|_{x_0,v_{\parallel,0}} + (v_\parallel - v_{\parallel,0})\left.\frac{\partial f_0}{\partial v_\parallel}\right|_{x_0,v_{\parallel,0}} \tag{39b}$$

Since

$$\frac{dx}{dt} = \frac{c}{B}\tilde{E}_\theta,\ \ \frac{dv_\parallel}{dt} = -\frac{|e|}{m_e}\tilde{E}_\parallel \tag{39c}$$

we immediately find:

$$\frac{\partial}{\partial t}\int d^3v\frac{\langle\delta f^2\rangle}{2} = -\langle\tilde{v}_r\delta n_e\rangle\left.\frac{\partial f_0}{\partial x}\right|_{x_0,v_{\parallel,0}} + \frac{|e|}{m_e}\langle\tilde{E}_\parallel\delta n_e\rangle\left.\frac{\partial f_0}{\partial v_\parallel}\right|_{x_0,v_{\parallel,0}} \tag{39d}$$

As before:

$$\delta n_e = \delta n_i - \rho_s^2\nabla_\perp^2\tilde{\phi} \tag{39e}$$

we thus obtain:

$$\begin{aligned}\frac{\partial}{\partial t}\int d^3v\frac{\langle\delta f^2\rangle}{2} =& -\left[\langle\tilde{v}_r\delta n_i\rangle - \langle\tilde{v}_r\rho_s^2\nabla^2\tilde{\phi}\rangle\right]\left.\frac{\partial f_0}{\partial x}\right|_{x_0,v_{\parallel,0}}\\ &+\frac{|e|}{m_e}\left[\langle\tilde{E}_\parallel\delta n_i\rangle - \frac{|e|}{m_e}\langle\tilde{E}_\parallel\rho_s^2\nabla_\perp^2\tilde{\phi}\rangle\right]\left.\frac{\partial f_0}{\partial v_\parallel}\right|_{x_0,v_{\parallel,0}}\end{aligned} \tag{40}$$

Here we ignore coherent (i.e. wave) non-adiabatic electron effects and hereafter drop the fourth term on RHS. Note that such correlations have been shown to potentially be significant in the context of intrinsic rotation[42]. Taking $\delta n_i = \chi_i(k,\omega)\tilde{\phi}_{k,\omega}$, where $\chi_i(k,\omega)$ is the ion guiding center susceptibility (n.b.: this tacitly ignores any ion trapping or granulations, but should include zonal shears), we find

$$\begin{aligned}\frac{\partial}{\partial t}\int d^3v\left\{\frac{\langle\delta f^2\rangle}{2\partial f_0/\partial x|_{x_0,v_{\parallel,0}}} + \langle V_\theta\rangle\right\} =& -\nu\langle V_\theta\rangle + \sum_{\mathbf{k}}k_\theta\mathrm{Im}\chi_i(\mathbf{k},k_\parallel v_{\parallel,0})|\tilde{\phi}_{\mathbf{k}}|^2\\ &+\sum_{\mathbf{k}}k_\parallel v_{\parallel,0}\mathrm{Im}\chi_i(\mathbf{k},k_\parallel v_{\parallel,0})\frac{f_0}{\partial f_0/\partial x|_{x_0,v_{\parallel,0}}}\end{aligned} \tag{41a}$$

or, collecting terms:

$$\frac{\partial}{\partial t}\int d^3v\left\{\frac{\langle\delta f^2\rangle}{2\partial f_0/\partial x|_{x_0,v_{\parallel,0}}}+\langle V_\theta\rangle\right\}=-\,\nu\langle V_\theta\rangle$$
$$-\sum_{\mathbf{k}}\frac{-k_\theta f_0'-k_\parallel v_{\parallel,0}f_0}{f_0'}\mathrm{Im}\chi_i(\mathbf{k},k_\parallel v_{\parallel,0})|\tilde{\phi}_{\mathbf{k}}|^2 \qquad (41b)$$

Here $v_{\parallel,0}$ is the hole speed, we have assumed f_0 is a Maxwellian, $\mathbf{k}$ refers to the modes excited by the structure (i.e. harmonics of the hole box size), and the hole ballistic frequency is $k_\parallel v_{\parallel,0}$. Note that Eq.(41b) states, absent zonal flows, that:

$$\partial_t\langle\delta f_e^2\rangle\sim-(\omega_{*e}-k_\parallel v_{\parallel,0})\mathrm{Im}\chi_i(\mathbf{k},k_\parallel v_{\parallel,0}) \qquad (41c)$$

so that $\omega_{*e}>k_\parallel v_{\parallel,0}$ is required for free energy accessibility. This, of course, the same as the familiar $\omega<\omega_*$ condition for instability in linear theory. That result requires that the structure motion release more energy due to radial scattering than it costs in $v_\parallel$ scattering. However, *unlike* linear theory, the dissipation which triggers growth is due to ions, i.e. $\mathrm{Im}\chi_i(\mathbf{k},k_\parallel v_{\parallel,0})<0$. This reflects the role of the ambipolarity constraint, discussed above, and implies that the regime of structure instability ($\mathrm{Im}\chi_i(\mathbf{k},k_\parallel v_{\parallel,0})$ large) is, in some sense, complementary to the regime of wave instability ($\mathrm{Im}\chi_i(\mathbf{k},k_\parallel v_{\parallel,0})$ small). This is similar to what we encountered for the case of CDIA. Also, clearly hole speed is a key parameter. $v_{\parallel,0}$ must be small enough so that $\omega_*>k_\parallel v_{\parallel,0}$, but not so small to drive $\mathrm{Im}\chi_i(\mathbf{k},k_\parallel v_{\parallel,0})\to 0$. Finally, we note that via the gyrokinetic Poisson equation, even electron holes will necessarily couple to, and drive, zonal flows.

As before, hole growth is nonlinear, i.e. explosive. A straightforward extimate which ignores zonal flow effects gives $\gamma\sim k_\parallel\Delta v_T(\omega_*-k_\parallel v_{\parallel,0})|\mathrm{Im}\chi_i(\mathbf{k},k_\parallel v_{\parallel,0})|$, where $\Delta v_T\sim\phi^{1/2}$. Holes growth will surely distort $\langle f\rangle$ eventually, thus vitiating some of the assumptions of this analysis. Note that the associated zonal flow growth can be calculated using the hole fluctuation driven Reynolds stress. Obviously, coupling to the zonal flow is a major player in the saturation of hole growth. Ref.[41] missed zonal flow effects, as it did not consider mesoscale fluctuation envelope variation. This omission is now rectified. Obviously, $\chi_i(k,\omega)$ must be calculated self-consistently, accounting for the zonal flow. Also, zonal flow shear will limit hole self-coherence times.

V. DUPREE-LENARD-BALESCU THEORY FOR MEAN EVOLUTION

In this section, we briefly sketch the essentials of the theory of $\langle f \rangle$ relaxation in a bath of turbulence which includes granulations and so has Kubo number $K \simeq 1$, rather than < 1, as nominally required for the applicability of quasi-linear theory. The main novel feature of the Dupree-Lenard-Balescu (DLB) theory is the appearance of a dynamical friction or drag effect, due to Cerenkov emission by phase space granulations or eddys. This effect is physically plausible in that intuitively put, turbulence at Kubo number $K \gtrsim 1$ behaves more like a soup of blobs or structures, rather than an ensemble of waves. In a soup of structures, each blob or hole will scatter off the others, and so leave a wake as it moves. This process of wake emission is easily described by dynamical friction effects. Just as structures find new ways to tap available free energy, dynamical friction can introduce new routes to relaxation, instability and transport. This discussion is intentionally very brief, as there have been other pedagogical treatments of the DLB theory published recently[18]. The interested reader is referred to these for the gory and gruesome details.

The essence of the DLB theory is to derive an equation for the 1-time, 2-point fluctuation correlation in phase space, $\langle \delta f(x_1, v_1, t) \delta f(x_2, v_2, t) \rangle$. This equation has the generic form:

$$\partial_t \langle \delta f^2 \rangle + T_{1,2}[\langle \delta f^2 \rangle] = P_{1,2} \tag{42}$$

Here $T_{1,2}$ refers to the two point evolution operator, including streaming, scattering and collisional dissipation. $T_{1,2}$ virtually *always* is calculated using a statistical closure of some form. The more interesting piece is the production term $P_{1,2}$. Since $df/dt = 0$ implies $d/dt \langle \delta f^2 \rangle = -\partial_t \langle f \rangle^2$, $P_{1,2}$ is obviously related to $\partial_t \langle f \rangle$ and thus to mean relaxation, transport, etc. For the 1D Vlasov prototype,

$$\partial_t \langle f \rangle = -\partial_v \langle \tilde{E} \delta f \rangle = -\partial_v [-D \partial_v \langle f \rangle + F \langle f \rangle] \tag{43}$$

Here D is analogous to the familiar quasilinear diffusion term, and F is the drag or dynamical friction term. F arises from the fact that

$$\delta f = f^c + \tilde{f} \tag{44}$$

i.e. the total fluctuation is the sum of a coherent (f^c) and incoherent ($\tilde{f}$) piece. The former is proportional to (i.e. coherent with) the electric field perturbation $\tilde{E}_{k,\omega}$, the latter is not. Consideration of the $x_-,\ v_- \to 0$ behavior of $T_{1,2}$ forces us to confront the existence of $\tilde{f}$. Thus:

$$-D\partial_v\langle f\rangle = \langle \tilde{E} f^c\rangle \tag{45a}$$

$$F\partial_v\langle f\rangle = \langle \tilde{E}\tilde{f}\rangle \tag{45b}$$

The incoherent correlation $\langle \tilde{f}(1)\tilde{f}(2)\rangle$ can be obtained from the total correlation $\langle \delta f(1)\delta f(2)\rangle$. $\tilde{f}$ is related to $\tilde{\phi}$ by

$$\epsilon(k,\omega)\tilde{\phi}_{k,\omega} = \int dv \tilde{f}_{k,\omega} \tag{46}$$

Thus, granulations resemble dressed macro-particles. It is possible for stationary turbulence to arise and persist in the absence of linearly unstable waves.

Collisionless coupling of two different species (i.e. electrons and ions) can occur via F and its dependence on ϵ. This induces dynamical friction. Just as two species interaction can lead to structure growth, dynamical friction can drive nonlinear instability, i.e. growth of δf^2 due to relaxation. Such growth has been observed in computer simulations[43]. Recently, the theory of collisionless dynamical friction was extended to include zonal flow effects[44]. The calculation of $P_{1,2}$ has also been used to estimate the efficiency of intrinsic rotation generation[45]. Several aspects of the results agree well with relevant experimental findings[46].

VI. CONCLUSION

This is a fascinating and active topic, which will only grow in importance and visibility. There are no meaningful final conclusions. Many interesting topics for future research might be suggested here. However, the authors would rather do these themselves! The reader is thus invited to think for himself or herself and beat us to the punch.

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[1] B. B. Kadomtsev, *Plasma Turbulence* (Academic Press, 1965).

[2] P. G. Drazin and W. H. Reid, *Hydrodynamic Stability* (Cambridge University Press, 2004).

[3] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon Press, 1959).

[4] G. K. Vallis, *Atmospheric and Oceanic Fluid Dynamics* (Cambridge University Press, 2006).

[5] G. I. Taylor, Phil. Trans. Roy. Soc. London., **A215**, 1 (1915).

[6] C. C. Lin, *The Theory of Hydrodynamic Stability* (Cambridge University Press, 1955).

[7] V. I. Arnold, Soviet Math. Dokl., **6**, 773 (1965).

[8] P. B. Rhines and W. R. Holland, Dynamics of Atmosphere and Oceans, **3**, 289 (1979).

[9] R. B. Wood and M. E. McIntyre, J. Asmos. Sci, **67**, 1261 (2010).

[10] P. H. Diamond, S. I. Itoh, K. Itoh, and T. S. Hahm, Plasma Phys. Control. Fusion, **47**, R35 (2005).

[11] S. M. Tobias, K. Dagon, and J. B. Marston, ApJ, **727**, 127 (2011).

[12] J. G. Charney and P. G. Drazin, J. Geophys. Res., **66**, 83 (1961).

[13] P. H. Diamond, O. D. Gurcan, T. S. Hahm, K. Miki, Y. Kosuga, and X. Garbet, Plasma Phys. Control. Fusion, **50**, 124018 (2008).

[14] O. D. Gurcan, P. H. Diamond, and T. S. Hahm, Phys. Plasmas, **13**, 052306 (2006).

[15] J. M. Ziman, *Principles of the Theory of Solids* (Cambridge University Press, 1979).

[16] I. B. Bernstein, J. M. Greene, and M. D. Kruskal, Phys. Rev., **108**, 546 (1957).

[17] T. H. Dupree, Phys. Fluids, **25**, 277 (1982).

[18] P. H. Diamond, S. I. Itoh, and K. Itoh, *Modern Plasma Physics Volume1: Physical Kinetics of Turbulent Plasmas* (Cambridge University Press, 2011).

[19] D. Lynden-Bell, Mon. Not. R. astr. Soc., **136**, 101 (1967).

[20] R. M. Kulsrud, *Plasma Physics for Astrophysics* (Princeton University Press, 2004).

[21] L. D. Landau, E. M. Lifshitz, and L. P. Pitaevskii, *Electrodynamics of Continuous Media* (Pergamon Press, 1981).

[22] T. H. Dupree, Phys. Fluids, **9**, 1773 (1966).

[23] H. L. Berk and B. N. Breizman, Phys. Fluids B, **2**, 2226 (1990).

[24] H. L. Berk and B. N. Breizman, Phys. Fluids B, **2**, 2235 (1990).

[25] H. L. Berk and B. N. Breizman, Phys. Fluids B, **2**, 2246 (1990).

[26] M. Lesur, Y. Idomura, K. Shinohara, X. Garbet, and the JT-60 Team, Phys. Plasmas, **17**, 122311 (2010).

[27] A. Vedenov and L. I. Rudakov, Dokl. Akad. Nauk SSSR, **159**, 767 (1964).

[28] A. Vedenov and L. I. Rudakov, Sov. Phys. Dokl., **9**, 1073 (1965).

[29] G. Laval and D. Pesme, Phys. Fluids, **26**, 52 (1983).

[30] S. I. Tsunoda, F. Doveil, and J. H. Malmberg, Phys. Rev. Lett., **59**, 2752 (1987).

[31] G. Darmet, P. Ghendrih, Y. Sarazin, X. Garbet, and V. Grandgirard, Communications in Nonlinear Science and Numerical Simulation, **13**, 53 (2008).

[32] M. Tagger, G. Laval, and R. Pellat, Nucl. Fusion, **17**, 109 (1977).

[33] H. Biglari, P. H. Diamond, and P. W. Terry, Phys. Fluids, **31**, 2644 (1988).

[34] V. A. Antonov, Vestnik Leningrad Gos. Univ., **7** (1962).

[35] J. Binney and S. Tremaine, *Galactiv Dynamics* (Pcineton University Press, 2008).

[36] M. D. Kruskal and C. R. Oberman, Phys. Fluids, **1**, 275 (1958).

[37] T. S. Hahm, Phys. Fluids, **31**, 1940 (1988).

[38] A. J. Brizard and T. S. Hahm, Rev. Mod. Phys., **79**, 421 (2007).

[39] B. B. Kadomtsev and O. P. Pogutse, *Reviews of Plasma Physics, Vol. 5*, edited by M. A. Leontovich (Consultants Bureau, 1970).

[40] A. Hasegawa and M. Wakatani, Phys. Rev. Lett., **50**, 682 (1983).

[41] P. W. Terry, P. H. Diamond, and T. S. Hahm, Phys. Fluids B, **2**, 9 (1990).

[42] C. J. McDevitt, , P. H. Diamond, O. D. Gurcan, and T. S. Hahm, Phys. Plasmas, **16**, 052302 (2009).

[43] R. H. Berman, D. J. Tetreault, and T. H. Dupree, Phys. Fluids, **28**, 155 (1985).

[44] Y. Kosuga and P. H. Diamond, Phys. Plasmas (2011).

[45] Y. Kosuga, P. H. Diamond, and O. D. Gurcan, Phys. Plasmas, **17**, 102313 (2010).

[46] J. E. Rice, J. W. Hughes, P. H. Diamond, Y. Kosuga, Y. A. Podpaly, M. L. Reinke, O. D. Gurcan, T. S. Hahm, A. E. Hubbard, E. S. Marmar, C. J. McDevitt, and D. G. Whyte, Phys. Rev. Lett., **106**, 215001 (2011).