Test of the Telegraph Equation for Transport Dynamics in Plasma

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(Received 26 July 2014 / Accepted 5 November 2014)

We have tested the telegraph equation for transport dynamics in magnetized plasma by comparing its results with experimental observations in the Large Helical Device [S. Inagaki *et al.*, Nucl. Fusion **53**, 113006 (2013)]. The telegraph equation includes a finite relaxation time for turbulence intensity in response to the changes in global plasma parameters. This model was applied to nondiffusive radial heat pulse propagation under the periodic modulation of the heating power. The model showed some success in reproducing the amplitudes of higher harmonics. However, the phase relation between the temperature gradient and heat flux was opposite to that in experimental observations.

Keywords: heat pulse propagation, telegraph equation, transport dynamics

DOI: 10.1585/pfr.10.1203002

1. Introduction

Recently, limitations have been recognized in conventional diffusive models of transport, which employ Fick's law to relate the temperature gradient and heat flux [1, 2]. In electron cyclotron heating (ECH) power modulation experiments in the large helical device (LHD), turbulent flux is changed before changes in local gradients occur [1]. In generalizing the relation between the temperature gradient and turbulent heat flux, one approach is to consider the finite time of relaxation (delay time) of turbulence in response to the changes in global plasma parameters; this approach is used in the K-ε model (or K-model) [3, 4]. Finite relaxation times are known to be important in analyzing the front dynamics of heat flux [5–8]. Here, we test the telegraph equation, which is a generalization of the diffusion equation and can be deduced in the presence of a finite time delay of turbulent heat flux. We apply the telegraph equation to the heat pulse propagation caused by periodic modulation of the heating power. The result is compared qualitatively with that from an LHD experiment [1]. However, the model cannot explain the phase relation in the hysteresis between the temperature gradient and the heat flux.

2. Telegraph Equation

For perturbations, the energy balance can be written as follows:

$$\frac{3}{2}n\frac{\partial \tilde{T}}{\partial t} = -\nabla \cdot \tilde{q} + \tilde{p}, \tag{1}$$

where n is the electron density and $\tilde{T}$, $\tilde{q}$, and $\tilde{p}$ are perturbations in the temperature, heat flux, and heat source, respectively. Here, we consider a region in which heating power is absent and the turbulent heat flux is induced by a temperature gradient. We assume that in the stationary state, the turbulent heat flux is related to the temperature gradient by

$$\tilde{q}_0 = -n\chi\nabla\tilde{T}, \tag{2}$$

where χ is the thermal diffusivity. The turbulence has its own relaxation time, thus the heat flux is assumed to converge to the stationary state with a finite relaxation time τ as follows:

$$\frac{\partial \tilde{q}}{\partial t} = -\frac{\tilde{q} - \tilde{q}_0}{\tau}. \tag{3}$$

Combining (1)-(3) gives the telegraph equation:

$$\frac{\partial^2 \tilde{T}}{\partial t^2} + \frac{1}{\tau}\frac{\partial \tilde{T}}{\partial t} - \frac{2\chi}{3\tau}\nabla^2\tilde{T} = 0, \tag{4}$$

where n and χ are assumed to be constant. The result (4) is a generalization of the conventional diffusion equation. In the limit of short relaxation times, $\tau \sim 0$, the diffusion equation is recovered. In the limit of τ longer than the oscillation period, the second term in Eq. (4) becomes negligible, and Eq. (4) approaches the wave equation without dispersion.

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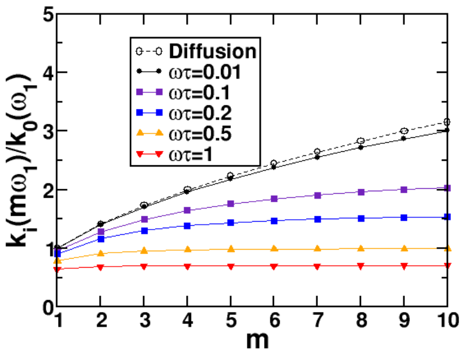

Fig. 1 Theoretical radial decay of higher harmonics of heat waves.

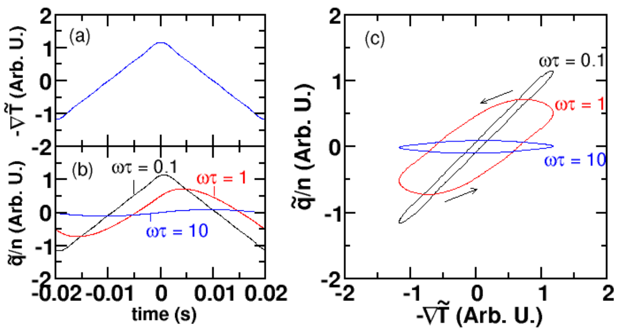

Fig. 2 Temperature gradients and heat flux computed from the telegraph equation.

We tested Eq. (4) by applying it to heat pulse propagation caused by periodic modulation of the heating power. We model the temperature perturbation by $\tilde{T} = A_0 \exp(ikx - i\omega t)$, where x is the distance from a reference radius. From the telegraph Eq. (4), we obtain the following:

$$k_r^2 - k_i^2 = \frac{3\tau\omega^2}{2\chi} = \gamma^2,\ k_r k_i = \frac{3\omega}{4\chi} = k_0^2,\ k_0 = \sqrt{\frac{3\omega}{4\chi}}. \tag{5}$$

The radial decay of the perturbation intensity is given as follows:

$$k_i = k_0 \sqrt{\sqrt{(\omega\tau)^2 + 1} - \omega\tau}. \tag{6}$$

Equation (6) shows that the radial decay rate (normalized to k_0) decreases as $\omega\tau$ increases; in contrast, the real part k_r/k_0 increases. This is natural because, in the limit of large $\omega\tau$, the equation converges to the wave equation without dispersion; the deformation of the waveform after radial propagation decreases as $\omega\tau$ increases. For the m-th harmonics, the radial decay rate is barely enhanced compared with the fundamental mode,

$$\frac{k_i^2(m\omega_1)}{k_0^2(\omega_1)} = m\omega_1\tau\left(\sqrt{m^2 + \frac{1}{(\omega_1\tau)^2}} - m\right). \tag{7}$$

This dependence is illustrated in Fig. 1. For $m > 1/\omega_1\tau$, the right-hand side of (7) only weakly depends on m. Compared with the standard diffusion model, this result is more consistent with observations on LHD and TJ-II [9, 10]. In those observations, the radial decay rate of amplitudes, k_i, for the 5th and 7th harmonics were barely enhanced, contrary to the predictions by the diffusion model.

As a caveat, the present model does not recover the phase relation observed in the LHD. We assumed $\nabla\tilde{T}$ and calculated $\tilde{q}$ at three different values of $\omega\tau$. Figure 2 demonstrates that the heat flux changes following the change in the gradient. The model gives a counterclockwise rotation in $(\nabla\tilde{T}, \tilde{q})$ space; however, the experimental results show clockwise rotation (Fig. 5 in [1]).

3. Summary

This is the first study of applying the telegraph equation to plasma transport. The equation includes a finite relaxation time for turbulence intensity in response to the changes in global plasma parameters. The model showed some success in reproducing the weak radial decay of higher harmonic components. However, the model could not reproduce the phase relation in hysteresis between the temperature gradient and heat flux. Further extensions of the model are needed to fully understand observations on the LHD.

This study is partly supported by a Grant-in-Aid for Scientific Research of JSPF, Japan (23244113, 23360414) and by the collaboration programs of NIFS (NIFS13KOCT001) and of the RIAM of Kyushu University and Asada Science Foundation.

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