

Scheme-independent error in entropy conservation in turbulent kinetic simulations

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Resonant interactions between particles and waves are relevant to a wide range of collisionless laboratory and space plasmas. Particle trapping in the troughs of electric potential is a fundamental kinetic nonlinearity, which leads to the formation of vortex-like structures in phase-space [1,2], especially in regimes of strong resonance. These structures have important impacts on wave stability, turbulent transport, intermittency, anomalous resistivity, heating, flows, etc. [3,4]

Accurate numerical simulation of such kinetic nonlinearities is challenging. Indeed, trapping involves the filamentation of phase-space (phase-space mixing). In the collisionless limit, numerical discretization replaces collisions to dissipate the smallest scales. A concern is that numerical dissipation and other simple models of collisions are artificial, and may impact the physics of interest. In this work, I show that the evolution of observables is surprisingly robust to changes of numerical treatment, which suggests that the details of small-scales dissipation are irrelevant. In particular, a systematic error of 15% in entropy conservation is found, regardless of the numerical treatment, without impacting observables.

In this work, as a paradigm for kinetic models in the presence of strong resonances, I restrict the analysis to one-dimensional ion-electron plasma with an initial velocity drift. The mass ratio is $m_i/m_e = 4$, the ion and electron temperatures are equal, and the initial velocity drift is $v_d = 4.2v_{T,i}$ (slightly above linear instability threshold), where $v_{T,i}$ is the ion thermal velocity. No collision operator is included. The simulation code is COBBLES [5], which is based on the Constrained-Interpolation-Profile, Conservative Semi-Lagrangian (CIP-CSL) scheme [6]. In this scheme, the evolutions of space- and velocity-integrals of the distribution function are computed from separate kinetic equations, along with the evolution of the distribution function itself. The implementation guarantees the local conservation of density. Fig.1 shows the redistribution of electrons and ions. Redistribution is due mainly to the evolution and interaction of phase-space structures (phase-space turbulence) [3].

Of common knowledge is that accurate simulations require careful treatment of conserved physical quantities, such as total mass, total energy and total entropy. For example, to obtain the turbulent steady-state accurately, spurious heating must be avoided. It is less

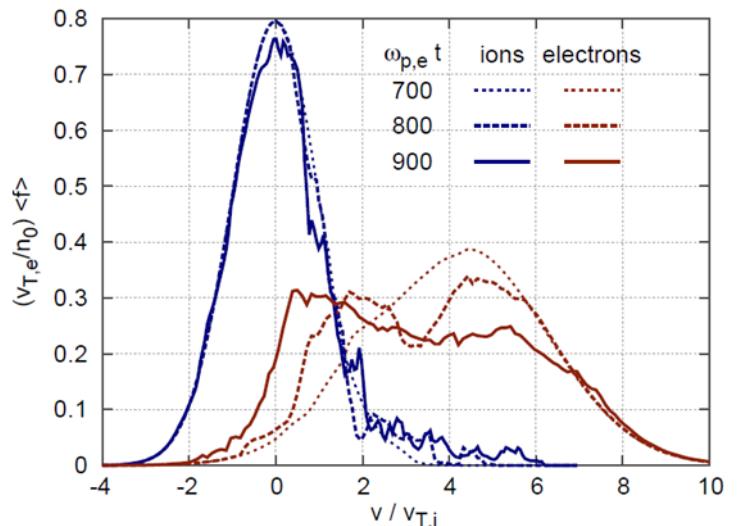


Fig.1. Snapshots of the velocity distributions.

known that the conservation properties strongly depend on the simulated physics. With the COBBLES code for two-species ion-acoustic turbulence, compared to one-species simulations of the dissipative bump-on-tail instability, the conservation of energy and entropy are degraded by several orders of magnitude. However, the quality of conservation is still satisfying, even in the long-time, fully nonlinear evolution of ion-acoustic turbulence. Indeed, the relative error in total energy remains below 0.1%.

Let me now focus on the entropy, which is trivially conserved in the physical model. The Fig.2 (a) shows the time-evolution of the error in electron entropy conservation, in COBBLES for four different numbers of grid points, and in a Particle-In-Cell (PIC) code, PICKLES, for two different numbers of particles. The error in total electron entropy conservation is well below 0.0001%, until phase-space filamentation occurs. Then, it quickly increases to 15%. Surprisingly, this error does not depend on the number of grid points, the number of particles, the time-step width (not shown in this figure), or even the type of numerical approach (Semi-Lagrangian or PIC). Although I don't have any clear explanation for this observation, one hypothesis is that the error in entropy conservation is actually a measure of infinitely small phase-space structures that are created during the nonlinear saturation.

A 15% error in entropy conservation is troubling. However, I didn't find any impact on the time-evolution of electric field, anomalous resistivity, or velocity distribution. As an example, Fig.2 (b) shows the time-evolution of electric field amplitude. In particular, let me compare two COBBLES simulations, with 256x1024 and 1024x1024 grid points. The time traces of electric field amplitude are almost perfectly overlapping, even though there is a significant discrepancy in the error in entropy conservation (14% and 15%).

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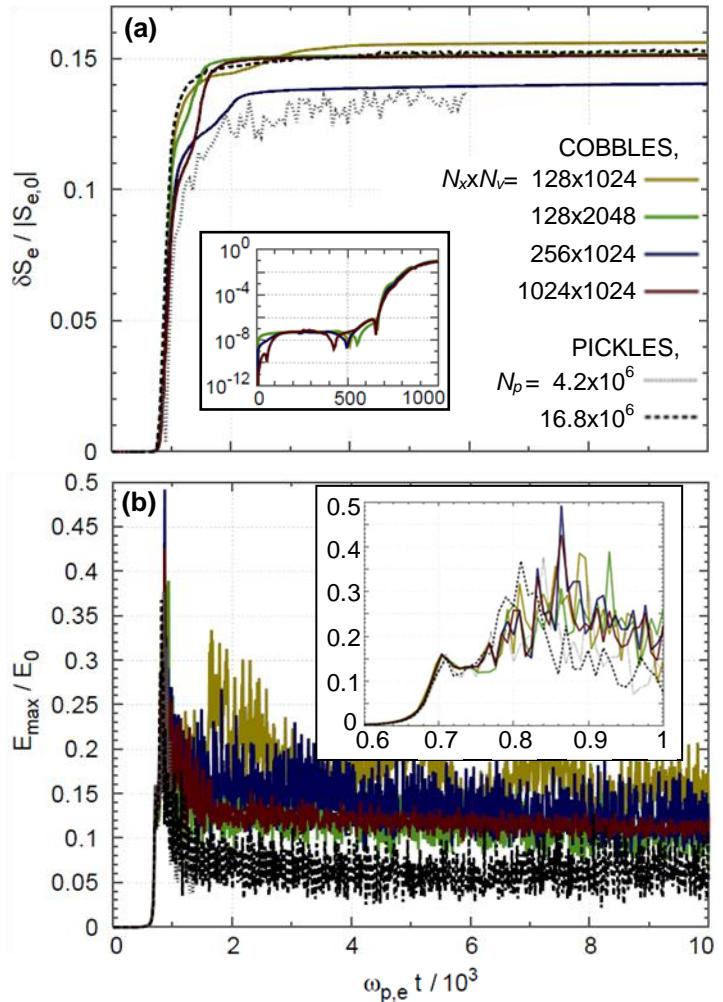


Fig.2. Time evolution of (a) error in entropy conservation and (b) electric field amplitude. Insets: zoom on a smaller timescale. The legend is shared.