Existence of Metastable Kinetic Modes

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The nonlinear evolution of resonantly driven systems, such as suprathermal particle driven modes in magnetically confined plasmas, is shown to strongly depend on the existence and nature of an underlying damping mechanism. When background resonant damping is present, subcritical states can take place. In particular, purely nonlinear steady-state regimes are found, whose destabilization threshold and saturation levels are calculated and validated using numerical simulations. This nonlinear behavior can be of relevance for acoustic modes in magnetically confined plasmas.

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In the present Letter, a nonlinear kinetic mechanism associated to wave-particle interaction is explored, which leads to purely nonlinear states, and, in particular, to metastable configurations. In addition to its academic interest, the nonlinear analysis of the stability of systems driven by kinetic resonant interactions is relevant to the understanding and control of magnetically confined burning plasmas. Indeed, in such systems, suprathermal particles such as fusion born alpha particles can resonantly excite deleterious waves, whereas abrupt events such as sudden crashes of the plasma central temperature (known as sawtooth crashes) can temporarily move the thermonuclear plasma away from equilibrium and hence make possible the access to purely nonlinear states. Similarly, it is reasonable to postulate the relevance of such an analysis for geophysical plasmas, where kinetic drive and resonance physics are known to take place [1]. The new metastability mechanism described here applies to waves which are both resonantly driven and damped. In tokamak plasmas, this situation applies to acoustic modes, such as β Alfvén eigenmodes and geodesic acoustic modes [2], which are of great importance because they can be used as energy channels to transfer the energy of fusion born alphas particles to the thermonuclear plasma. The latter waves are resonantly driven by suprathermal ions and subject to thermal ion Landau damping. In the following, a reduced model is used to study such a dynamics.

In tokamak plasmas, theoretical and experimental linear analysis have been conducted to determine the destabilization threshold of modes kinetically driven by suprathermal particles. Although purely nonlinear configurations are known to exist for some magnetohydrodynamic modes (e.g., neoclassical tearing modes [3]), the nonlinear modification of the stability threshold of resonantly driven modes and the resulting existence of nonlinear modes have not been investigated to our knowledge. To conduct this analysis, a 1D multispecies electrostatic bump-on-tail model is developed. The latter model, with some variations, has been widely used to study the nonlinear behavior of kinetically driven modes, and it can be proved to be equivalent to the description of modes driven by suprathermal particles in a tokamak, when transforming to appropriate variables and for sufficiently smooth geometry gradients [4]. It has been shown to appropriately describe the experimental nonlinear behavior of energetic particle driven modes such as toroidal Alfvén eigenmodes or fishbone modes, in particular, close to their linear stability threshold [5,6]. Here, two Vlasov equations are used ($i = 1, 2$) along with the Poisson equation:

$$\partial_t F_i + v\partial_x F_i + (e_i E/m_i)\partial_v F_i = -\nu_i[F_i - F_{i\mathrm{eq}}(v)], \quad (1)$$

$$-\partial_x E = \sum_{i=1,2} e_i \int dv[F_i - F_{i\mathrm{eq}}(v)], \quad (2)$$

where the physical quantities are normalized using arbitrary reference quantities for the density, temperature, charge, and mass n, T, e, m (such that frequencies are normalized by $\omega_p = \sqrt{ne^2/m\epsilon_0}$ with ϵ_0 the dielectric constant in vacuum, lengths by $\lambda = \sqrt{\epsilon_0 T/ne^2}$, velocities by $v_t = \sqrt{T/m}$, distribution functions by v_t/n, the electric field by $\sqrt{\epsilon/nT}$, and finally charges and masses, respectively, by e and m). Here, the equilibrium distribution of the first species $F_{1\mathrm{eq}}$ is taken to present a thermal *bulk* with a higher energy bump designed to provide resonant *excitation*, $F_{1\mathrm{eq}} = (n_b/\sqrt{2\pi}v_{\mathrm{tb}})\exp(-[v/v_{\mathrm{tb}}]^2/2) + (n_e/\sqrt{2\pi}v_{\mathrm{te}})\exp\{-[(v - v_0)/v_{\mathrm{te}}]^2/2\}$. Thus, in the absence of the second species, Eqs. (1) and (2), correspond to the standard electrostatic bump-on-tail problem [7] with collisions (where the bulk and bump collisionalities are the same $\nu_b = \nu_e = \nu_1$) appropriate for the description of electron driven Langmuir waves in a background of fixed ions. In this study, a second evolving species with a thermal distribution function $F_{2\text{eq}} = (n_d/\sqrt{2\pi}v_{\text{td}}) \times \exp(-[v/v_{\text{td}}]^2/2)$ and a collisionality $\nu_d = \nu_2$ is added to include a tunable resonant *damping*.

In earlier studies, the relevance of taking into account a background damping was recognized but simply added to the standard bump-on-tail problem as a fixed damping rate $-\gamma_d$ ($\gamma_d > 0$). Such models with a single species [8,9] and a fixed damping [4] have been studied extensively in the nonlinear regime, both theoretically and numerically [10], in the monochromatic (single k wave number), perturbative (small bump), cold-bulk ($\omega \gg kv_{\text{tb}}$) limit, for which the bump induced linear drive can be put in the form of a γ_d-independent excitation rate γ_e ($\gamma_e > 0$). In this limit, when a linearly unstable mode (i.e., with $\gamma_e > \gamma_d$) grows, resonant driving particles are known to get trapped and to bounce inside the mode phase-space structure [11] with a characteristic bounce frequency $\omega_{\text{Be}} = \sqrt{e_e E k/m_e}$, which reduces the resonant drive. More precisely, if ν_e, $\omega(v_0/v_{\text{te}}) \ll \omega_B$ such that decorrelation processes are weak, trapping is dominant and resonant drive is reduced by the factor $\alpha\nu_e^* = \alpha(\nu_e/\omega_{\text{Be}})$ with $\alpha \approx 2.0$. Thus, if $\gamma_d \lesssim \nu_e$, steady-state saturation can be reached when $\alpha\nu_e^*\gamma_e = \gamma_d$, which has been confirmed numerically [11,12]. We now want to investigate the effect of a self-consistent resonant damping. Similarly as for the drive, one can expect a nonlinear reduction of damping. Writing ν_e^* and ν_d^* the nonlinear reduction factors to apply to the drive and damping rates, respectively (in case similar reduction factors exist), it is reasonable to postulate the existence of subcritical states (defined as states with $\gamma_e < \gamma_d$) if

$$\nu_e^*\gamma_e > \nu_d^*\gamma_d. \tag{3}$$

In the following, the existence of such a subcritical activity is demonstrated. Among the various subcritical states, metastable states (defined as subcritical steady-state states) are analyzed in detail. Their existence conditions and saturation levels are derived. Note that the relationship $\nu_e^*\gamma_e = \nu_d^*\gamma_d$, with the definition of ν^* given above is degenerate in the mode amplitude and does not provide saturation *a priori*.

For simplicity, the limit described above is used in the remainder of the Letter. More precisely, writing $\omega_{\text{pb}} = \sqrt{n_b e_b^2/m_b}$ the normalized bulk plasma frequency, it is assumed that $\omega_{\text{pb}} \gg kv_{\text{tb}}$, ν_b, ν_e, ν_d, and that $n_b \gg n_e$, n_d such that the linear growth and damping rates γ_e and γ_d are perturbative (γ_e, $\gamma_d \ll \omega_{\text{pb}}$). The linear resolution of Eqs. (1) and (2), leads to the dispersion relation

$$-k^2 = \sum_{s=b,e,d} \frac{\omega_{\text{ps}}^2}{v_{\text{tb}}^2} Y\left(\frac{\omega + i\nu_s - \delta_{\text{se}} k v_0}{\sqrt{2}kv_{\text{ts}}}\right), \tag{4}$$

with $Y(x) = (1 + x)Z(x)$ and Z the plasma dispersion function, and $\delta_{\text{se}} = 1$ for $s = E$, 0 otherwise. With the cold bulk and perturbative approximation, it is classical that such dispersion relation returns a wave of frequency $\omega_0^2 = \omega_{\text{pb}}^2$ at the leading order, whereas linear stability is given at the higher order by the linear growth rate $\gamma_L \approx \gamma_e - \gamma_d - \nu_b$ with

$$\gamma_s = \left| 0.5 \frac{\omega_{\text{pb}}}{k^2} \frac{\omega_{\text{ps}}^2}{v_{\text{ts}}^2} \text{Im} Y\left(\frac{\omega_{\text{pb}} - \delta_{\text{se}} k v_0}{\sqrt{2}kv_{\text{tb}}}\right) \right|. \tag{5}$$

Classically, the bulk collisionality is taken to be small ($\nu_b \ll \gamma_e$, γ_d, ω_{pb}) in the linear analysis.

The model has been implemented numerically. Simulations are performed using an initial perturbation of the form $F_1(x, v, t = 0) = F_{1\text{eq}}(1 + \alpha\cos(kx))$. The following parameters are kept fixed: $k = 0.3$, $e_s = 1.0$, $m_s = 1.0 (s = b, e, d)$, $n_b = 1.0$, $v_{\text{tb}} = 0.3$, $n_e = 0.03$, $v_{\text{te}} = 1.0$, $v_0 = 4.5$, $v_{\text{td}} = 2.5$ and agree with the perturbative ($n_e \ll n_b$) and cold-bulk ($kv_{\text{te}} \ll \omega_{\text{pe}} \sim 1$) requirements. The remaining parameters (n_d, m_d/m_e and the collisionalities ν_e and ν_d) are kept free. In particular, collisionalities are chosen independently of the other parameters, since our intention is to study a paradigmatic model, which can cover various types of dissipative effects. An example of such a simulation is displayed in Fig. 1 representing a stability diagram in the (ω_{pd}^2, ν_e) space for fixed values of $m_d/m_l = 2.0$ and $\nu_d = 0.04$. The diagram is topologically similar to the one simulated earlier with the fixed damping model in Ref. [10] where the (γ_d, ν_e) plane was used [recall that $\gamma_d \propto \omega_{\text{pd}}^2$ from Eq. (5)], and returns analogous saturation regimes: steady-saturation regimes, excitation-relaxation regimes, chaotic regimes, and damped modes. Some reduced subcritical activity confined to the chaotic region is found, which is reminiscent of the subcritical states found in Ref. [12], but metastable regimes are absent.

Let us now increase the ratio ν_e^*/ν_d^* where $\nu_s^* = \nu_s/\omega_{\text{Bs}}$ to favor the emergence of metastable states following the intuition of Eq. (3). In Fig. 2, the parameters $m_d/m_l = 0.5$ and $\nu_d = 0.005$ are used. Obviously, the simulations return a large number of steady-state as well as chaotic subcritical regimes. Next, the former regime is analyzed.

As explained earlier, Eq. (3) does not directly lead to saturation. It can be noted that such a relation assumes low collisionality (negligible background dissipation ν_b and $\nu_s \ll \omega_{\text{Bs}}$, $s = e$, d). Several reasons justify avoiding this constraint and to consider $\nu_s = O(\gamma_e, \gamma_d, \omega_{\text{Bs}})$ ($s = e$, d). First, even for $\nu_b \ll \gamma_e$, γ_d, the nonlinear reduction of the drive and damping can give the leading order to various background dissipation effects, which justifies keeping them in the final balance. Second, the nonlinear trapping reduction is valid for $\nu_s \ll \omega_{\text{Bs}}$ ($s = e, d$). Since we want to investigate the boundaries of the mode stability

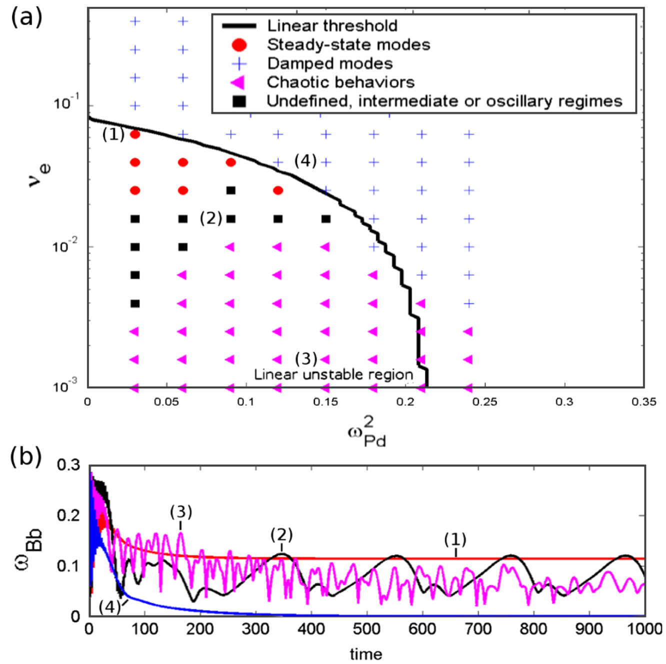

FIG. 1 (color online). Nonlinear states obtained when varying $(\omega^2_{\rm pd}, \nu_e)$ for fixed values of $m_d/m_l = 2.0$, $\nu_d = 0.04$ and for a large initial perturbation $\omega_{\rm Bb}(t = 0) = 0.2$. Four behaviors are distinguished, illustrated with the time evolution of their amplitude (given as a function of $\omega_{\rm Bb}$) for four particular sets of parameters: $(\omega^2_{\rm pd}, \nu_e) = (0.03, 0.063)$ in (1), $(0.09, 0.016)$ in (2), $(0.15, 0.002)$ in (3), and $(0.15, 0.040)$ in (4). The linear threshold is calculated numerically from Eq. (4).

diagram for a wide range of parameters, we do not wish to rely on this condition *a priori*. When $\nu_s \sim \omega_{\rm Bs}$, collisions are strong enough to compete with trapping such that linear rates are recovered (it is the "quasilinear regime"). It is reasonable to average these two limits to obtain the regimes corresponding to intermediate values of $\nu_s^* = \nu_s/\omega_{\rm Bs}$, hence to use a reduction factor of the form $\alpha\nu_s^*/(1 + \alpha\nu_s^*)$ asymptotically equal to $\alpha\nu_s^*$ for small values of ν_s^* and to 1.0 for large values of this parameter. In this way, the degeneracy of Eq. (3) is removed. Accordingly, the evolution of the mode amplitude (expressed here in terms of $\omega^2_{\rm Bb} \propto E$) close to a saturation point (where $d/dt \ll \omega_{\rm Bs}$, $s = e, d$) follows Eq. (6),

$$\frac{d\omega^2_{\rm Bb}}{dt} = \omega^2_{\rm Bb}[\mathcal{E}(\omega_{\rm Bb}) - \mathcal{D}(\omega_{\rm Bb})], \tag{6}$$

with

$$\mathcal{E}(\omega_{\rm Bb}) \equiv \frac{\alpha\nu_e}{\omega_{\rm Be} + \alpha\nu_e}\gamma_e, \qquad \mathcal{D}(\omega_{\rm Bb}) \equiv \frac{\alpha\nu_d}{\omega_{\rm Bd} + \alpha\nu_d}\gamma_d + \nu_b. \tag{7}$$

Steady-state saturation is possible if positive solutions exist for the second order equation in $\omega_{\rm Bb}$, $d/dt = 0$. When $\gamma_L \equiv \gamma_e - \gamma_d - \nu_b > 0$, that is in the linear unstable region, it always has a positive solution. In the subcritical region, positive solutions exist under the conditions that

$$\gamma_{\rm NL} \equiv \nu_e^*(\gamma_e - \nu_b) - \nu_d^*(\gamma_d + \nu_b) > 0 \tag{8}$$

and

$$\Delta \equiv (\gamma_{\rm NL})^2 - 4\nu_e^*\nu_d^*\nu_b|\gamma_L| > 0. \tag{9}$$

More exactly, in the subcritical case, Eq. (6) has two positive solutions $\omega^{\pm}_{\rm Bb} = (\bar\gamma_{\rm NL} \pm \sqrt{\bar\Delta})/\nu_b$, where $\bar\gamma_{\rm NL} \equiv \gamma_{\rm NL}/\omega_{\rm Bb}$ and $\bar\Delta \equiv \Delta/\omega^2_{\rm Bb}$ are amplitude independent. It is easy to see that $\mathcal{E}' - \mathcal{D}'$ is positive for the lower solution and negative for the upper one, which is consequently the only stable solution. Finally, when $\nu_e^* > \nu_d^*$ and $\nu_b \ll \gamma_e$, γ_d, Eqs. (8) and (9) allow for the existence of metastable modes, and Eq. (8) is close to our previous intuition. This confirms that the driving and damping species properties involved in the ν_s^* parameters need to be considered for stability analysis. In particular, in the bump-on-tail problem (where collisionalities and masses are the relevant properties), the roles of mass and density (which appeared in $\omega_{\rm ps}$ in the linear analysis) are decorrelated nonlinearly. Interestingly, the present model allows for the existence of steady-state modes in the entire linearly stable region, which is in agreement with the fixed γ_d model. It can be noted that both the consideration of the background dissipation or the one of the large collisions for resonant particles would have been sufficient to remove the degeneracy of Eq. (3), but that none of the two effects kept alone allows for steady-state regimes in the whole linear region.

In Fig. 2 the region where steady-state modes are possible is displayed [given by Eqs. (8) and (9)] using γ_e and γ_d defined by Eq. (5). In this figure, the initial magnitude is $\omega_B(t = 0) = 0.2$, and was chosen large enough to avoid strong modification of the diagram for larger $\omega_B(t = 0)$. The simulated metastable regimes are obviously in good agreement with the analytic nonlinear threshold.

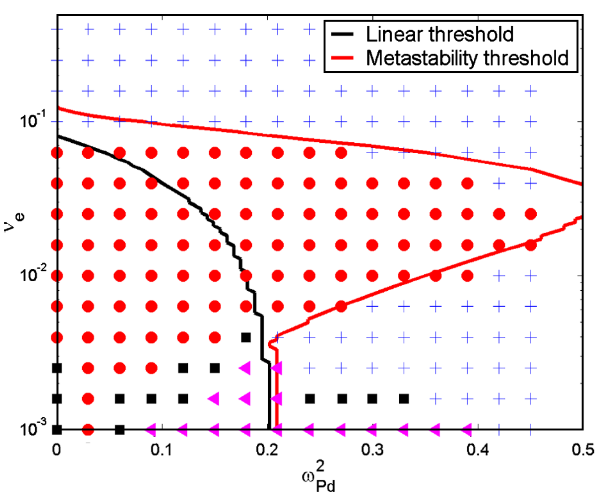

FIG. 2 (color online). Nonlinear states obtained when varying the parameters $(\omega_{\rm pd}, \nu_e)$, for fixed values of $m_d/m_l = 0.5$, $\nu_d = 0.005$, and for the initial perturbation $\omega_{\rm Bb}(t = 0) = 0.2$. The classification of states is the same as in Fig. 1.

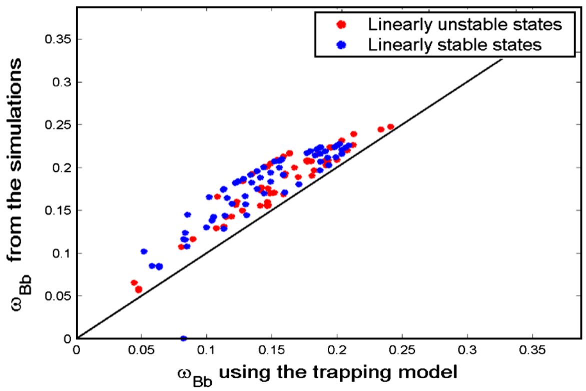

FIG. 3 (color online). Simulated steady-state saturation levels compared to prediction, obtained for $m_d/m_l \in \{0.5, 2.0\}$, $\nu_d \in \{0.001, 0.005, 0.01\}$, and full scans in $\nu_b \in [10^{-3}, 0.1]$, $\omega_{\rm pd}^2 \in [0.0, 0.35]$ similar to Fig. 1.

More quantitatively in Fig. 3, simulated saturation levels are compared with the upper solution of Eq. (3), for a large range of parameters, with various reduction factors. Again, the agreement is good and shows the relevance of the calculation of Eq. (6), not only for subcritical states, but also for catching the modification of the saturation levels of linearly unstable modes induced by a self-consistent damping.

Finally, it remains to understand how to trigger such metastable modes. Equation (6) indicates that it is sufficient to exceed the lower saturation solution, $\omega_{\rm Bb}(t=0) \geq \omega_{\rm Bb}^-$.

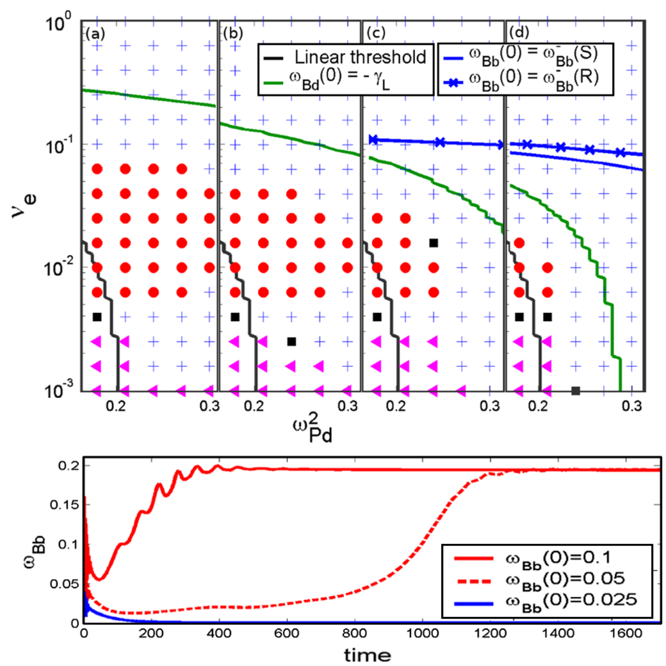

FIG. 4 (color online). Stability diagrams corresponding to $m_d = m_l = 0.5$, $d = 0.005$, obtained for various magnitudes of the initial perturbation: $\omega_{\rm Bb}(t=0) = 0.1$ in (a), $\omega_{\rm Bb}(t=0) = 0.05$ in (b), $\omega_{\rm Bb}(t=0) = 0.025$ in (c). The amplitude time evolution of one particular set of parameters (encircled) is given to illustrate the decay of the mode at low initial perturbation.

Remarkably, this suggests that very small triggering amplitudes ($\omega_{\rm Bb}^- \to 0$ close to marginality) can lead to modes of large amplitudes ($\propto \gamma_{\rm NL}$ close to marginality). When using a refined quasilinear regime of growth rate $[1-(1/8)\times(\omega_{\rm Bs}/\nu_s)^4]\gamma_s$ (derived from Ref. [13]) instead of γ_s, the conclusion is even more striking. To verify this, the occurrence of modes is compared with $\omega_{\rm Bb}^-$ (with and without the refinement) for different values of the initial perturbations in Fig. 4. Obviously, the criterion $\omega_{\rm Bd}(t=0) > -\gamma_L$ which results from the validity condition of Eq. (6) (that is, the predominance of nonlinear trapping over the mode evolution time scale) has to be taken into account additionally. It plays the major role for the present set of simulation parameters.

In summary, the present Letter shows the possible existence of a subcritical activity for resonantly driven and damped modes. In particular, it is explained how steady-state metastable modes can take place, and criteria for their existence are analyzed. When applied to tokamak acoustic β Alfvén eigenmodes, it can be shown that $\nu_d^* \sim \nu_e^* \sim 0.01$ [14], which means that metastable configurations are realistic. Although the present analysis is limited to a perturbative treatment, it puts forward an important physics to consider in experiments if one hopes to use acoustic modes as energy channel from fusion born alpha particles to the thermal plasma, via external excitation by antennas.

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[1] A. J. Klimas, J. Geophys. Res. **95**, 14 905 (1990).
[2] F. Zonca, L. Chen, and R. A. Santoro, Plasma Phys. Controlled Fusion **38**, 2011 (1996).
[3] R. Fitzpatrick, Phys. Plasmas **2**, 825 (1995).
[4] H. L. Berk *et al.*, Phys. Plasmas **6**, 3102 (1999).
[5] M. K. Lilley, B. N. Breizman, and S. E. Sharapov, Phys. Rev. Lett. **102**, 195003 (2009).
[6] H. L. Berk *et al.*, Nucl. Fusion **46**, S888 (2006).
[7] T. O'Neil, Phys. Fluids **8**, 2255 (1965).
[8] G. Manfredi, Phys. Rev. Lett. **79**, 2815 (1997).
[9] M. V. Medvedev, P. H. Diamond, M. N. Rosenbluth, and V. I. Shevchenko, Phys. Rev. Lett. **81**, 5824 (1998).
[10] R. G. L. Vann *et al.*, Phys. Plasmas **10**, 623 (2003).
[11] H. L. Berk and B. N. Breizman. Phys. Fluids B **2**, 2226 (1990).
[12] M. Lesur, Y. Idomura, and X. Garbet, Phys. Plasmas **16**, 092305 (2009).
[13] H. L. Berk, B. N. Breizman, and M. Pekker, Phys. Rev. Lett. **76**, 1256 (1996).
[14] C. Nguyen *et al.*, "Nonlinear Modification of the Stability of Fast Particle Driven Modes" [Plasma Phys. Controlled Fusion, EPS Special Issue (to be published)].